Swiss Bank 3 Walkthrough

Once again, we can seek the integer either by constructing it digit by digit, or by seeking the whole answer. We give both methods:

Digit by digit construction:

If an integer has the same number of digits when it is multiplied by 9, it must start with a 1. Its next digit must be either a 0 or a 1. However 11… would give 99… when multiplied by 9 which wouldn't work, as putting a 9 in front would give 911… So the only possibility is that the first two digits are 10:

First two digits are 10

and the product must begin with a 9. This means the last digit is 9. When we multiply by 9 we find 9*9 = 81 so the last digit of the product must be 1, carrying 8 towards the next digit and

The last two digits are 19.

Continuing in this manner we have 9 x 1 + carry over 8 = 7 carry 1, 9 x 7 + 1 = 4 carry 6 so that

The last 4 digits are 4719.

Continuing the backwards construction, we have

9 x 4 + 6 (carried over) = 42 = 2 carry 4

9 x 2 + 4 (carried over) = 2 carry 2

9 x 2 + 2 (carried over) = 0 carry 2

9 x 0 + 2 (carried over) = 2

giving 2022 for the preceding 4 digits. We are given these and the previous 4 also, 943820224719. Continuing,

9 x 4 + 3 (carried over) = 9 (the leftmost digit we have) carrying over 3

9 x 9 + 3 = 4 (carry 8)

9 x 4 + 8 = 4 (carry 4), so that

The previous two digits are 44

When we are given …8988764044943820224719. Continuing,

9 x 9 + 7 = 8 carry 8 (leftmost digit we have)

9 x 8 + 8 = 0 carry 8

9 x 0 + 8 = 8, so that

The previous two digits are 80.

We then have …61797752808988764044943820224719.

9 x 1 + 7 = 6 carry 1

9 x 6 + 1 = 5 carry 5

9 x 5 + 5 = 0 carry 5

The previous two digits are 05.

We now have …112359550561797752808988764044943820224719

9 x 1 + 1 = 0 carry 1

9 x 0 + 1 = 1

The previous two digits are 10 which joins up with the 1st two digits.

We now have a possible solution, 44 digits long. However, instead of terminating after 44, we could have continued the process, going round the loop once more. This would duplicate the 44 digit string giving us an 88 digit solution.
1011235955056179775280898876404494382022471910112359550561797752808988764044943820224719

Construction of entire integer

To construct an integer with the required multiplication property, call it x and suppose it has k digits, with a last digit N. Then removing the last digit is equivalent to subtracting N and dividing by 10. We then put N at the front which is adding N x 10^{k-1}. We deduce that

$9x= (x-N)/10 + N(10^{k-1})$

Or $90x = x-N +N(10^k)$

And solving for x

$x = N(10^k-1) / 89$

This requires $10^k - 1$ to be divisible by 89. Using "Fermat's Little Theorem", since 89 is prime, this happens for k = 88. We know N = 9 giving

$x = 9(10^{88}-1)/89 =$
1011235955056179775280898876404494382022471910112359550561797752808988764044943820224719

By inspection, we spot the smaller solution 10112359550561797752808988764044943820224719.

Note $(10^{88}-1) = (10^{44}-1)(10^{44} + 1)$ and $10^{44} + 1$ = 1000000……001 which on multiplication appends a 44 digit number to itself. So provided 89 divides $10^{44} - 1$ and not $10^{44} + 1$, we have a smaller solution.