Solution to Noughts and Crosses Logic 1

The end solution is the same with or without the extra information, although there are two possible games leading to the same position in the latter case.

Hint 1 points the solver to the strategy in Os and Xs to help progress after B1. Hint 2 and Hint 3 give extra information to help progress after B1 and C2 respectively. It is possible, but harder, to solve the problem without the extra information. The first solution below uses the hints, while the second solution does not. Both solutions begin the same:

1 A2 is empty (given)

- There are 4Xs and one empty square in the final position so there must be 3 or 4 Os (because of the blank square A2 there cannot be 5 Os).
- As there is a blank square, the game must have ended with a win.
- If X won then there would be 3 Xs in a row. Each X is connected to another with a knight's move, and there would have to be at least 5 Xs in the final position. So O must have won the game, and played last, with 4 Os in the final position. X played the first move.

The central square is not connected to any other by a knight's move so is not X. As it is not blank, we deduce:

2 B2 is O

B1 was the only bad move of the game. As X lost, X must have played a bad move. So:

3 B1 is an X

- After X's first move, which is the first move of the game, O's first move is in the same column.
- In noughts and crosses, if the first move is in a corner (here X), the second move (here O) must be in the centre square (HInt 1). To see this, consider what happens if the second move is elsewhere:

```
X1 O2 __     X1 __ O2     X1 __ __     X1 O4 X3
O4 __ __     O4 __ __     O4 X5 O2     __ __ __
X3 __ X5     X3 __ X5     X3 __ __     X5 __ O2
```

- The centre square is not in the same column as any corner so, as O does not play any bad moves, it follows that X's first move is not in a corner.
- So X's first move must be B1, A2, B2, C2, or B3. However:

a) B1 on the 1st move is not a "bad move". So X's first move was not B1 (B2 clue).
b) A2 is empty. So X's first move was not A2.
c) B2 is O. So X's first move was not B2.
d) C2 on the first move would require O's first move to be B2 as A2 is empty. So X's first move was not C2 (Hint 2).

So:

4 B3 is X [To deduce this without using Hint 2, see below]

- Row B is known to be XOX. The other 2 Xs must be in the corners in order to provide each row B X with an X a knight's move away. There are 6 ways of arranging the 2 Xs, however if they are both in column 1 or 3 then there will be be a vertical line of 3 Xs
- The only possible winning lines for O are the row C and the 2 diagonals (since columns 1 and 3 and row B have X in them and column 2 and row A have the empty A2 in them). The 2 Xs cannot both be on the bottom row as this would block all the possible winning lines for O, so the 3 possible configurations are:

```
  1           2           3

X _ X       X _ O       O _ X

X O X       X O X       X O X

O O O       O O X       X O O
```

In all of these C2 is O so:

5 C2 is O

Hint 3 implies that B1 was the second X placed.

Note also that the How to Play says that where a 3 in a line is possible it must be completed - if the opponent does not block a 3 in a line when they can then the game will finish early and e.g. the Xs will not all have an X within a knight's move.

For position 2 the game has to go:

```
_ _ _       _ _ O       _ _ O       _ _ O       _ _ O

_ _ X       _ _ X       X _ X       X O X       X O X

_ _ _       _ _ _       _ _ _       _ _ _       X _ _
```

which does not work because position 2 does not have an X in C1

For position 3 the game has to go:

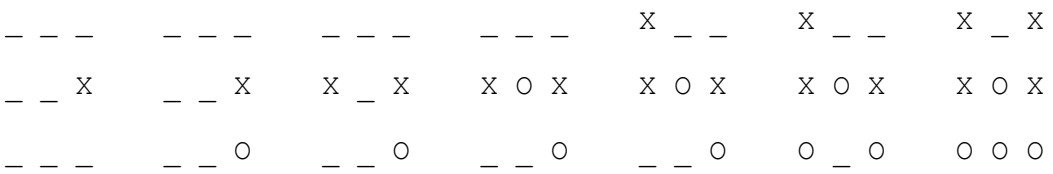
```
_ _ _       _ _ _       _ _ _       _ _ _       X _ _

_ _ X       _ _ X       X _ X       X O X       X O X

_ _ _       _ _ O       _ _ O       _ _ O       _ _ O
```

which does not work because position 3 does not have an X in A1

So:

6 The correct answer is position 1

The game goes:



Solution without using the extra information in the hints:

Deductions 1-3 as above, followed by the proof that X's first move is either C2 or B3. Then

4. If the game began X:C2, O would have to reply O:B2 (must be in the same column and A2 is empty). At some point X plays at B1, but not on the 2nd, as this would not lose by force. The other two X-moves must be corner squares by the knight's-move constraint. If A3, B1 and C2 are all Xs, the 4th X would have to be either A1 or C3 by the knight's moves. But this would block all possible lines of 3 Os, so the game does not begin X:C2, O:B2, X:A3.

5. If the game started X:C2, O:B2, X:C3, it would have to continue O:C1, X:A3, O:B3, X:B1 and this does not end in an O-win. Note that if X does not block at A3, the game would terminate too soon after the O:A3 move required by the rules. Similarly, if the game started X:C2, O:B2, X:C1, it would continue O:C3, X:A1, O:A2, but we know A2 must be an X.

6. If the game started X:C2, O:B2, X:A1, the other two Xs would have to be on A2 and C3, leaving the winning O-line C1-B2-A3. To achieve this on the 4th move, O could not play at C1 or A3 on the 2nd move, and so would have to try O:B3. But the game would finish X:B1, O:C1, X:C3(??), O:A3. But then the bad move would be C3, not B1. We conclude that the first move is not X:C2, and from above, the first move must be X:B3, and so **B3 is an X**. O's first move must be either A3 or C3.

7. As B1 and B3 are Xs, the other 2 Xs are in corners. It follows that **C2 is an O.** (This step can be omitted).

8. If the game starts X:B3, O:A3, the other Xs must be on B1, C3 and either A1 or C1. B1,B3,C1,C3 leaves no winning O-line, so the Xs must be B1,B3,A1,C3. The start X:B3, O:A3, X:C3 is a bad move, losing to O:A1 (X:A2, O:C1), so is impossible. The start X:B3, O:A3, X:B1, O:B2 does not terminate in the required manner. Finally, after the start X:B3, O:A3, X:A1 threatens X:B1, so O must immediately play on C1 or B2 and X must then break the required pattern. It follows that the game must commence X:B3,O:C3, and **C3 is an O.** The possible 2nd moves for X are at A1,B1,C1 or A3.

9. The start X:B3,O:C3,X:C1 threatens X:B1 and X:B2 and only the moves O:B1 or O:B2 avoid losing. Either of these lead to an O:B1, which is forbidden. The start X:B3,O:C3,X:A3 is a bad move losing to O:C1 so is impossible.

10. The start X:B3,O:C3,X:B1 leads by force to O:B2, X:A1, O:A3 followed by X:A3,O:C2, and **is a possible solution**. X:B1 is a bad move. The start X:B3,O:C3,X:A1 requires the other Xs to be on B1 and A3, giving **the same final position.** A second possible game would be X:B3,O:C3,X:A1,O:B2, X:B1(??) O:C1, X:A3, O:C2. We conclude that **A1 is an X, A3 is an X, C1 is an O.**

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I hope this is clear! Please send any queries, suggestions or corrections to A_O_D.