Solution to Hermione's Arithmancy OWL

Background data: HP was born on 31st July 1980. His disciplinary hearing was in Court 10. He came of age in 1997. He was the only person ever to survive the Avada Kedavra curse. His Gringott's vault number was 687. In his year were the 2 twins Padma and Pavarti Patil. There were 5 boys in his dormitory, Harry, Ron, Neville, Seamus and Dean. There are 4 Houses at Hogwarts and 2 pubs in Hogsmeade. There are 3 magical schools in Europe, with Durmstrang and Beauxbatons. There are 7 players on a Quidditch team, and scoring with the Quaffle earns 10 points. 4 times platform 9¾ is 39. There are 32 black squares on any chess board. The giant Acromantula spider Aragog had 8 eyes (and presumably 8 legs). Molly & Arthur Weasley had 7 children. From a London phonebox dial 999 for the Emergency services or 62442 ("MAGIC") for the Ministry of Magic. Dumbledore is credited with discovering 12 uses of dragon's blood.

Putting all these together (with the numbers in increasing order) gives 2 columns of numbers which need to be squared and added to make the third column:

1	2	5
2	3	29
7	4	73
8	5	193
10	7	1061
31	10	1621
687	12	472993
999	32	998017
1980	39	7908409
62442	1997	3899003413

The small numbers in the 3rd column are easy. 5 = 4 + 1, so **(1, 2, 5)** is one triple. 29 = 25 + 4, so **(2, 5, 29)** is another. 73 = 64 + 9, so **(8, 3, 73)** is a third. This leaves:

1	2	5
2	3	29
7	4	73
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10	7	1061
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193 has to be grouped with one of [7, 10] and one of [10, 12] as any number over 30 is too large and 4 and 7 in the second column are both too small. 10 doesn't work because 93 isn't a square, so it must use 12.
Indeed 144 + 49 = 193, so **(7, 12, 193)** is a 4th triplet.

The two years each square to roughly 4 million. No other number comes close enough, so we must have the 5th triplet **(1980, 1997, 7908409**). Indeed, this is true if one checks with a calculator.

Clearly 62442 squared is involved with 3899003413. Thinking about the last digit, the other square has to end with a 9. The only remaining 2nd column number for which this holds is 7, so we have the 6th triplet **(62442, 7, 2899003413).** This leaves:

1	2	5
2	3	29
7	4	73
8	5	193
10	7	1061
31	10	1621
687	12	472993
999	32	998017
1980	39	7908409
62442	1997	3899003413

999 x 999 is nearly 1,000,000 so must match with 998017. Again considering the last digit, it must be added to a square ending 6 to give a number ending with a 7, giving **(999, 4, 998017).** Similarly, 687 squared ends in a 9 so requires a square ending 4 giving **(687, 32, 472993).**

The last two triplets are **(10, 39, 1621)** and **(31, 10, 1061).**

Incidentally, if you're interested in the mathematical background, look for Fermat's Christmas theorem online.