Factorials Logic Puzzle Walkthrough

[Note: for anyone wishing a hint rather than the full solution, it is recommended that you consider prime factors rather than multiplying out the factorials. The position of the numbers can be deduced in the order: (1), 4, 5, 11, 7, 10, 21, 24, 6, 3, 9, 23, 15, 8, 25, 12, 14, 18, 16, 20, 13, 22, 2, 19, 17.]

1	A2	A3	A4	A5
B1	B2	B3	B4	B5
C1	C2	C3	C4	C5
D1	D2	D3	D4	D5
E1	E2	E3	E4	E5

We are told (A2)! + 1 = $C2^2$, (C2)! + 1 = $B5^2$. As $25^2 = 625$ and 6! = 720, the only possibilities are $4! + 1 = 5^2$, $5! + 1 = 11^2$. We deduce that

A2=4, C2=5, B5=11.

1	4			
				11
	5			

We learn (D1)! + 1 = (5^{th} column sum)². Now the largest a column sum could be is 21+22+23+24+25 =115; $115^2 = 13225$. Now 7! = 5040, so 8! Is already too large, and we only have to check up to 7. The only possibility is $7! + 1 = 5041 = 71^2$. So Column 5 sums to 71 and

D1=7.

We learn that the product of the bottom row equals B1!, and that B1 is as large as it could be. Now 25 x 24 x 23 x 22 x 21 < 11!, so B1 < 11. Is 10 possible?

Now $10! = 2^8 \times 3^4 \times 5^2 \times 7$, a total of 15 prime factors. If this is going to equal the product of 5 numbers, each number must on average have 3 prime factors. Counting the factors of the numbers 1-25 we have

4 Factors:	16 24
3 Factors:	8 12 18 20
2 Factors	4 6 9 10 14 15 21 22 25
1 Factor	1 2 3 5 7 11 13 17 19 23

There are only 4 numbers with 3 factors, so we need at least one with 4 factors to make our total of 15. We could subdivide the 15 factors as 4 3 3 3 2 or 4 4 3 3 1 or 4 4 3 2 2.

16 and 24 use up 7 factors of 2 so that if both are used the only possibility is 16 x 24 x 18 x 25 x 21. Nothing else works. We conclude that 10! can be written as the product of the bottoms row, but 11! cannot. As B1 is the largest possible, we have

B1=10.

1	4			
10				11
	5			
7				

We learn that the 5th column has the product 11 x 12 x 13 x 14 x 15 = 15!/10!. We also know that the 5th column sums to 71, and we know it must contain 11 and 13 and either 14 or 21 (as 7 is elsewhere) and either 15 or 20 (as 5 and 10 are used). The possibilities are:

11 13 14 15 12 (total 65)

11 13 14 20 9 (total 67)

11 13 21 15 8 (total 68)

11 13 21 20 6 (total 71)

The last set has the correct total. The 5th column and 5th row intersect at E5. (21) is the intersection of the two sets (16, 24, 18, 15, 21) and (6, 11, 13, 20, 21) and so

E5=21.

				6, 13, 20	
1	4				
10				11	
	5				
7					
				21	16,18,24,25

We learn that 3 of the factorials (1, 2, 6, 24) are in corners. This requires

E1=24, A5=6.

				13, 20	
1	4			6	
10				11	
	5				
7					
24				21	16,18,25

We are told the product of the 1st column is a factorial. C1=3 would give 7!, while C1=24 would give 8!. But 24 has already been used and so **C1=3.** We are also told the product of the diagonal E1-A5 is C4!. We know that the product must be less than 11!, so that looking at the available numbers either C4=8 or C4=9. If C4=8, then 8!=24 x 6 x D2 x C3 x B4 or D2 x C3 x B4 = 5 x 7 x 8. As 5 &10 are unavailable and 20 is in the 5th column, this is not possible. So **C4=9,** and D2 x C3 x B4 = 5 x 7 x 8 x 9.

14 is needed as 7 & 21 are unavailable, and 15 is needed as 5, 10, 20 are unavailable. So (D2, C3, B4) = (12, 14, 15) in some order.

				13, 20	
1	4			6	
10				11	
3	5		9		
7					
24				21	16,18,25

We learn that the product of all numbers other than column 3 is 21!. This means the product of column 3 is 22 x 23 x 24 x 25. We are also told that the same is true of row 2. This fixes the intersection **B3=23.** Then B2 x B4 = 120. The only available multiple of 5 is 15, so that recalling what we know about the E1-A5 diagonal **B4=15, B2=8.** The product of the 3rd column is 22 x 23 x 24 x 25. Then considering factors of 5 fixes **E3=25,** and factors of 3 and 7 require **C3=12, D2=14.** The factors of 11 fix (A3, D3) = (2, 22) in some order.

		2, 22		13, 20	
1	4			6	
10	8	23	15	11	
3	5	12	9		
7	14				
24		25		21	16,18

We learn the product of column 2 is a factorial. This fixes **E2=18, E4=16** and column 2 has the product 8!. (In the original version we are then told that the product of the middle row and 16 is $(6!)^2$.) In the revised version, we are told that C5 divides the factorial of every number in the 4th column. Each requires **C5=20 & so D5=13.** The unused numbers are the large primes 17 & 19.

		2, 22	17, 19	
1	4			6
10	8	23	15	11
3	5	12	9	20
7	14			13
24	18	25	16	21

Finally, we learn that D4 divides D3! and vice versa. This requires **D3=22 & A3=2.** Also D4 does not divide A4!, so that **D4=19** and **D3=17**.

1	4	2	17	6
10	8	23	15	11
3	5	12	9	20
7	14	22	19	13
24	18	25	16	21

The welcome whistle blows, and it's the end of another hard day's work at the Factorial Factory. Hope you enjoyed it.