Sums of Neighbours – Solution and Comments

We label the boxes as follows:

A1	**A2**	**A3**	**A4**	**A5**
B1	**B2**	**B3**	**B4**	**B5**
C1	**C2**	**C3**	**2**	**C5**
D1	**D2**	**D3**	**D4**	**D5**
E1	**E2**	**E3**	**E4**	**C5**

C4=2

We are told that 3 numbers equal the product of all the numbers, and that all the numbers are integers. As we know that at least one number is bigger than one, this can only happen if the product of all the numbers is zero, and so we know

C3=0, B4=0, D4=0.

By considering square C4 we see that C5+0+0+0=2, so

C5=2.

Now let B3 be b. Then from square B4 we have b+2=-(A4+B5)= -A5 from square A5. Then from square B5 we know B5=A5+2+0 and so B5=-b. Then from square B4 we know A4+2+b-b=0 or

A4=-2.

Square A3 now tells us that b=b-2+A2 so that

A2=2.

Square B3 requires that

B2=0.

The diagram now looks like

	2	b	-2	-b-2
	0	b	0	-b
		0	2	2
			0	

Now we can repeat all the arguments we have used in the top half in the lower half to infer

E2=2, E4=-2, D2=0

and extend the solution to

	2	b	-2	-b-2
	0	b	0	-b
		0	2	2
	0	c	0	-c
	2	c	-2	-c-2

Square C5 now tells us c=-b. Then square C3 requires C2=-2 and then square C2 requires C2=C1

C2=-2, C1=-2

which gives us a second clue, namely that the lower two corners E1+E5= -14. Combining squares E2 and E4 we have c+E1+0=2 and c+E5+0=-2 so 2c+E1+E5=0 or 2c=14 and c=7 and so b=-7. The rest follows by arithmetic giving the solution

9	2	-7	-2	5
7	0	-7	0	7
-2	-2	0	2	2
-7	0	7	0	-7
-5	2	7	-2	-9

Incidentally, the Sporcle software seems to ignore the "-" character in some circumstances, so that some implementations of this puzzle would suffer from the flaw that when the solution is -5, typing "5" would be treated as correct. As a result we have insisted on two-character answers, which is a slight nuisance for the solver. If anyone has a better way of getting round this, please mail us.

Aside for Mathematicians: If we regard each number as an unknown, we have 25 simultaneous linear equations in 25 unknowns, which can be written as a matrix equation M**x**=0, where **x** is the vector of unknowns and M is a 25 x 25 matrix. These equations always have the trivial solution **x**=0, but there may be additional solutions if M has an eigenvector with zero eigenvalue. In fact, for this problem there are two linearly independent eigenvectors, so that the general solution to the problem can be written, for arbitrary numbers a and b,

a-b	a	b	-a	-a-b
-b	0	b	0	-b
-a	-a	0	a	a
b	0	-b	0	b
a+b	a	-b	-a	b-a

Thus every possible solution for a 5x5 grid has the same X-pattern of zeroes and so the first clue in the puzzle, which fixes 3 of them, isn't actually strictly necessary, nor is the constraint that the numbers be integers. However they do allow the puzzle to be solved more readily by forwards logic.

Further note for Applied Mathematicians: The "sum of the neighbours" constraint can be regarded as a finite difference approximation to the Helmholtz equation for u(x,y) $u_{xx} + u_{yy} + k^2u = 0$ with u = 0 on the boundary and a suitable k. Thus the solution eigenvectors correspond to approximations to the normal modes of vibration for a square drum, $u = \sin(mx)\sin(ny)$, which tallies with the lines of zeroes in the eigenvectors when $a=0$ or $b=0$.