N. H. BINGHAM: Research Interests, 2009

I am a probabilist, who is also interested in analysis, statistics and mathematical finance. The ongoing theme of most of my work is *limit theorems in probability*. This is inseparable from *tail behaviour* – that is, the (small) probabilities of very large, very small or otherwise *extreme values*. The prototypical results here are the Law of Large Numbers (LLN) (or rather Laws, as there are many such results) – the Law of Averages of the man in the street; the Central Limit Theorem (CLT) – the Law of Errors of the physicist in the street; the Law of the Iterated Logarithm (LIL), specialised enough to be generally known only to probabilists, which splits the difference between the two; and the Large Deviation Principle (LDP), which focusses on exponential rates of decay of very small probabilities of extreme events. Things are simplest in the classical setting – independent and identically distributed (iid) real-valued random variables. But one may generalise independence to weak dependence (in various ways) – the point being that the LLN is about *cancellation*, and even weakly dependent errors tend to cancel. One may also generalise to *random vectors*, in many – or infinitely many – dimensions, to random matrices, etc. See the bibliography on my CV for details of my papers in this area.

The kind of mathematical analysis relevant to this area of probability involves the theory of it regular variation. I am the co-author (with C. M. Goldie and J. L. Teugels – BGT below) of the standard work in the area (*Regular variation*, CUP, 1987/89).

My current work focusses on three main areas.

Topological Regular Variation.

This theory is joint work with Adam (Dr A. J.) Ostaszewski of the Mathematics Department, London School of Economics. See Adam's web page there for details of preprints etc. ("[BOst]", in the LSE C-DAM series), his or my CV for published papers, and our forthcoming book ("Bostaszewski") for a monograph synthesis.

While BGT (even 22 years on) still gives fairly full coverage of most of regular variation, it contains two glaring gaps, one at each end:
1. At the beginning, the theory is developed in tandem for the two classical cases, the measurable case and the Baire case (that is, for functions having the Baire property). The authors raise, and leave open, the question of finding the natural common generalisation of these two cases (neither of which

contains the other). This has now been done. A by-product is the new insight that, while the measurable case (which came earlier) has traditionally been regarded as the primary case, it is in fact the Baire case which is primary (the measurable case can be treated in tandem with it, bitopologically – by switching from the Euclidean topology to the density topology).
2. At the end, the question is raised of generalisation beyond the real-valued case – to d dimensions, to infinitely many dimensions, to topological groups etc. Much recent work in the area of extreme-value theory (EVT) has been devoted to such generalisations. The theory of topological regular variation provides a natural framework here.

Stationary processes and prediction theory.

This line of work is in collaboration with Professor Akihiko Inoue of Hiroshima University and Dr Yukio Kasahara of Hokkaido University, Sapporo, Japan.

For simplicity, let us keep here to discrete time, where the theory of stationary processes is usually called that of (stationary) time series (TS). Traditionally, this subject can either be treated using *time domain* methods (autocorrelation function), or *frequency domain* methods (spectral density or measure) – though with wavelets, one can handle both together.

For many technical purposes, the autocorrelation function $\gamma = (\gamma_n)_{n=0}^{\infty}$ is less convenient than the *partial autocorrelation function* (PACF) $\alpha = (\alpha_n)_{n=0}^{\infty}$. The theory of the PACF has been developed in detail in a number of papers since 2000 by Inoue and Kasahara (separately or together). The theory of *orthogonal polynomials on the unit circle* (OPUC – the subject of the recent two-volume monograph by Barry Simon) is highly relevant here. Our current work uses OPUC, function theory on the unit disk and other areas to take the study of the PACF and its applications further.

Lévy and other models in mathematical finance.

My work in this area is joint with Rüdiger Kiesel of the University of Ulm.

The benchmark model of mathematical finance is the Black-Scholes model, where the driving noise process is Brownian motion. More general models are needed, for example to include jumps. Lévy processes (in one or many dimensions – dimension d is needed for a portfolio of d assets) are useful here; so too are more general diffusions (again, in one or many dimensions). Our current work here focusses on *multivariate elliptic processes.* NHB, 2.4.2009