**3.** *a)* Sketch the 3-state "competing risk" Markov model having two absorbing states. Use arrows to show the possible transitions between states.

*b)* List the three assumptions regarding transition probabilities made by the standard actuarial model in *a)*.

*c)* An insurance firm uses the 3-state competing risk model to model the lives of policyholders insured against death by sneezing. Let state 0 denote 'active policyholder', state 1 denote 'death by sneezing' and state 2 denote 'death by other means'.

In a population of $N$ customers, let $(d_i, v_i)$ denote the recorded statistic on the $i$th life where $d_i \in \{0, 1, 2\}$ indicates the state occupied by the $i$th life at the end of a waiting time $v_i$ in state 0. All lives begin in state 0.

*i)* Starting from your assumptions in *b)*, derive the likelihood for $(d_i, v_i)$ as a function of the model parameters. Hence write down the likelihood of the full data set and derive the form of the maximum likelihood estimates of the transition intensities between states.

*ii)* Hence or otherwise, derive expressions for the transition probabilities $p_{01}(t)$ and $p_{02}(t)$ where $p_{0j}(t)$ denotes the probability of belonging to state $j$ at time $t$ given the chain is in state 0 at time 0.

*iii)* Suppose the following summary statistics are obtained from a study of 50 lives. Sixteen are observed to die by sneezing, 24 die by other means and the total waiting time in state 0 is 20 years. Show that,

$$\widehat{p}_{01}(\log 2) = 0.3, \qquad \widehat{p}_{02}(\log 2) = 0.45,$$

where $\widehat{p}_{ij}(t)$ denotes the maximum likelihood estimate of the transition probability from state $i$ to state $j$ at time $t$.