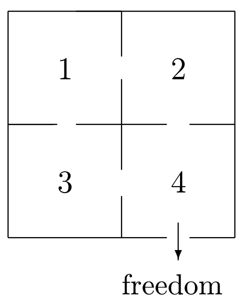
M3S4/M4S4: Applied probability: 2007-8
Assessed Coursework 2
Distributed: Wednesday, 27 February
Due: Wednesday, 12 March (before 4pm)

1. Matrices with non-negative entries in which the rows sum to 1 are sometimes called *stochastic matrices*. The transition matrices of Markov chains are stochastic matrices.

   (a) Show that stochastic matrices have at least one eigenvalue equal to 1.

   (b) Hence show that if P is a stochastic matrix, then so is P^n .

2. A rat runs through the following maze:



   The rat starts in a given cell and at each step it moves to a neighbouring cell (chosen with equal probability from those available, independently of the past). It continues moving between cells in this way until it escapes to the outside. Assume that the outside (freedom) is denoted state 0 (and can only be reached from cell 4). We assume that once the rat has escaped it remains escaped forever.

   (a) Determine the transition matrix; draw the transition diagram and find the communicating classes associated with this chain.

   (b) Find the stationary distribution if it exists.

   (c) Find m_i: the expected number of steps taken to escape given that the rat starts in cell i, for $i = 1, 2, 3, 4$.

   Hint: condition on the first step (i.e. take similar initial approach used to evaluate expected duration of the gambler's ruin problem).