M3S4/M4S4: Applied probability: 2007-8

Problems 2: Point Processes

1. Data are recorded showing the time of occurrence of every third event from a Poisson process of rate λ. Recording started at the time of occurrence of an event:

   (a) What is the distribution of T_i, $i = 1, \ldots, n$, the time between the $(i-1)$th and ith event?

   (b) What is the distribution of W_n, the total time to the nth event?

2. Given a Poisson process with rate λ, what is the distribution of the time between the ith and jth $(j > i)$ events?

3. Data are recorded showing the time of occurrence of every second event from a process in which the inter-event time is $\chi^2(1)$. What is the distribution of the recorded process?

4. Which of the following probability statements about a continuous time point process are true, where $X(t)$ = number of events at time t and W_n = time the nth event occurs?

   (a) $\mathrm{P}(W_n > t) = \mathrm{P}(X(t) < n)$.

   (b) $\mathrm{P}(W_n > t) = \mathrm{P}(X(t) \leq n)$.

   (c) $\mathrm{P}(W_n > t) = \mathrm{P}(X(t) < n + 1)$.

   (d) $\mathrm{P}(W_n \leq t) = \mathrm{P}(X(t) > n)$.

   (e) $\mathrm{P}(W_n \leq t) = \mathrm{P}(X(t) \geq n)$.

5. Customers arrive at a bank according to a Poisson process at a mean rate of 10 per minute. 60% of the customers wish to withdraw money (type A), 30% wish to pay in money (type B), and 10% wish to do something else.

   (a) What is the probability that more than 5 customers arrive in 30 seconds?

   (b) What is the probability that in 1 minute, 6 type A customers, 3 type B customers, and 1 type C customers arrive?

   (c) If 20 customers arrive in 2 minutes, what is the probability that just one wants to carry out a type C transaction?

(d) What is the probability that the first 3 customers arriving require only to make a type A transaction?

(e) How long a time will elapse until there is a probability of 0.9 that at least one customer of type A and one of type B will have arrived? (you will need to solve this numerically).

6. A bank opens at 10.00am and customers arrive according to a non-homogeneous Poisson process at a rate $10(1+2t)$, measured in hours, starting from 10.00.

   (a) What is the probability that two customers have arrived by 10.05?

   (b) What is the probability that 6 customers arrive between 10.45 and 11.00?

   (c) What is the probability that more than 50 customers arrive between 11.00 and 12.00?

   (d) What is the median time to the first arrival after the bank opens?

   (e) By what time is there a probability of 0.95 that the first customer after 11.00 will have arrived?

7. Give a compound Poisson process $X(t)$ in which situations arise according to a Poisson process with rate λt and for which the number of events which occur at the ith situation is a random variable Y_i, with mean μ and variance σ^2, derive the variance of the number of events which will have occurred by time t.

8. Show that the index of dispersion for a non-homogeneous Poisson process is 1.

9. Suppose that cars arrive at an airport according to a Poisson process with rate λ, and that the number of people in a car has a binomial distribution $Binomial(4, p)$. What are the mean and variance of the number of people who have arrived by time t?

10. A person makes shopping expeditions according to a Poisson process with rate λ. The number of purchases he makes is distributed according to a geometric distribution $G_1(p)$. What are the mean and variance of the number of purchases made in time t?

11. Derive the index of dispersion for a compound Poisson process in which Y, the number of occurrences at each compound event, is distributed as

    (a) $G_0(p)$.

    (b) Poisson(μ).