M3S4/M4S4: Applied probability: 2005-6
Problems 4: Random Walks

1. Two players, A and B, play a series of independent games, in which A has a probability p of winning and a probability $q = 1 - p$ of losing. If A begins with £j and B with £$(a - j)$, show that the match is certain to end eventually.

2. Two players, A and B, play a series of independent games, in which each start with £500. In each game, they toss a fair coin, with A winning £1 when heads come up and B winning £1 when tails come up. For how long is the series expected to run?

3. A particle moves according to a simple random walk with

$$\mathrm{P}(Z = 1) = 0.8; \qquad \mathrm{P}(Z = -1) = 0.2.$$

   (a) What is the probability that the particle is more than 10 units from the origin after 25 steps?

   (b) Find a range of positions within which the particle will be with probability 0.95 after 100 steps.

4. In an unrestricted random walk starting at the origin, the ith step, Z_i, has distribution $\mathrm{P}(Z_i = 2) = p$ and $\mathrm{P}(Z_i = -1) = q = 1 - p$.

   (a) Find the mean and variance of Z_i.

   (b) Hence find the mean and variance of X_n, the position of the particle after n steps.

   (c) Derive the probability distribution of X_n. (Use the binomial distribution method outlined during the lectures)

   (d) If $p = 1/3$, find the values of
   (i) $\mathrm{E}(X_{20})$, (ii) $\mathrm{var}(X_{20})$, (iii) $\mathrm{P}(X_{20} = 0)$, (iv) $\mathrm{P}(X_{20} = 1)$.

   (e) When $p = 1/6$, find the approximate value of $\mathrm{P}(-70 < X_{180} < 70)$.

5. Given a simple random walk with $p = q = 1/2$, what is the probability that the first return to the origin occurs at the

   (a) 4^{th} step?

   (b) 10^{th} step?