1. (a) The maps are monotonically increasing if $x > 1$ and montonically decreasing if $x < 0$, so the non-wandering sets must be contained in $[0.1]$. The map f satisfied $f([0,1]) = [0,1]$, so hence $\Lambda_f = [0,1]$. The map g does not map $[0,1]$ to itself, for instance $g(5/12) \notin [0,1]$ and $\lim_{n\to\infty} |g^n(5/12)| = \infty$, so $5/12 \notin \Lambda_g$.

(b) (i) Let $\Delta_0 := [0, \frac{1}{3}]$, $\Delta_1 := [\frac{1}{3}, \frac{2}{3}]$, and $\Delta_2 := [\frac{2}{3}, 1]$. Let Σ_3 denote the set of (half) infinite sequences in the three symbols $\{0,1,2\}$ and $h : \Sigma_3 \to [0,1]$ be given by $h(\omega_0\omega_1\ldots) = \overline{\bigcap_{i=0}^{\infty} f^{-1}(\mathrm{Int}(\Delta_{\omega_i}))}$, with "Int" denoting the interior Since $|\Delta_{\omega_0\omega_1\ldots\omega_j}| \leq \frac{1}{3}|\Delta_{\omega_0\omega_1\ldots\omega_{j-1}}|$ it follows that $\lim_{j\to\infty} \Delta_{\omega_0\omega_1\ldots\omega_j} \in [0,1]$. By construction h is not invertible due to ambiguity on boundary points between labelling intervals, for instance $h(02222222\ldots) = h(1222222\ldots) = 1/3$. h is continuous if we endow Σ_3 with the metric $d(x,y) := \sum_{i=0}^{\infty} \delta_i(x,y)/4^i$ where $x = x_0x_1\ldots$ and $y = y_0y_1\ldots$ with $x_k, y_k \in \{0,1,2\}$ for all $k \in \mathbb{N}$ and $\delta_i(x,y) = 0$ if $x_i = y_i$, and $\delta_i(x,y) = 0$ otherwise. By construction then $f \circ h(\omega) = h \circ \sigma(\omega)$ for any $\omega \in \Sigma_3$, where $\sigma : \Sigma_3 \to \Sigma_3$ denotes the shift $\sigma(\omega_0\omega_1\omega_2\ldots) = \omega_1\omega_2\ldots$.

(ii) For g we follow a similar construction, with $\Delta_0 := [0, \frac{1}{3}]$, $\Delta_1 := [\frac{4}{9}, \frac{5}{9}]$, and $\Delta_2 := [\frac{2}{3}, 1]$ and we observe now that for each fixed j, the labelling domains $\Delta_{\omega_0\omega_1\ldots\omega_j} := \overline{\bigcap_{i=0}^{j} f^{-1}(\mathrm{Int}(\Delta_{\omega_i}))}$ are disjoint. Hence, the limit points $h(\omega)$ uniquely depend on ω so that h is invertible on Λ_g.

2. (i) The topology is such that each open neighbourhood of Σ_3 contains a cylinder $C_{\alpha_0\ldots\alpha_j} := \{\omega = \omega_0\omega_1\ldots \in \Sigma_3 \mid \omega_i = \alpha_i,\ 0 \leq i \leq j\}$ that are the open balls (monotonically and uniformly shrinking in size as $j \to \infty$).

Density of periodic orbits is demonstrated by the fact that every cylinder set contains a periodic sequence. Namely, A is transitive as A^2 contains no zero entries. Hence given $\alpha_0\ldots\alpha_j$, for every choice of $\alpha_{j+2} \in \{0,1,2\}$ there exists an $\alpha_{j+1} \in \{0,1,2\}$ such that $\alpha_0\ldots\alpha_j\alpha_{j+1}\alpha_{j+2}$ is an admissible subsequence of elements of $\Sigma_{3,A}$ (the set of A-admissibe subsequences of Σ_3). This in turn implies that given any $\alpha_0\ldots\alpha_j$ there exists α_{j+1} such that the period sequence $\overline{\alpha_0\ldots\alpha_j\alpha_{j+1}} \in C_{\alpha_0\ldots\alpha_j,A} := C_{\alpha_0\ldots\alpha_j} \cap \Sigma_{3,A}$.

To prove topological mixing, let V, W be open with $C_{\alpha_0\ldots\alpha_j} \subset V$ and $C_{\beta_0\ldots\beta_k} \subset W$, then for all integer values of $p > 1$ there exist $\alpha_{j+1},\ldots,\alpha_{j+p} = \beta_0$ such that $\alpha_0\ldots\alpha_j\alpha_{j+1},\ldots,\alpha_{j+p}$ is an admissible subsequence of $\Sigma_{3,A}$ so that $\alpha_0\ldots\alpha_j\alpha_{j+1}\ldots\alpha_{j+p}\beta_1\ldots\beta_k$ is admissible as well and thus for all $p > 1+j$ $f^p(V) \cap W \neq \emptyset$.

(ii) We first note that admissible sequences are made up of blocks "0", "10", and "20", concatenated in arbitrary order. Given any element of a cylinder set $C_{\alpha_0\ldots\alpha_j,A}$, we can find another element of this cylinder whose sequence eventually differs but in the worst case, for the element $x = \alpha_0\ldots\alpha_j\overline{0}$ we can only create a sequence that has a mismatch with the tail of this sequence at best only every other digit. In other words, $\max_{y\in C_{\alpha_0\ldots\alpha_j,A}, p\in\mathbb{Z}^+} d(f^p(x), f^p(y)) = \sum_{i=0}^{\infty}(1/4)^{2i} = 16/15$.