

(2)

$$\tilde{G}_A : \Omega_A \rightarrow \Omega_A ; \quad (\tilde{G}_A(w))_i = w_{i+1}$$

Definition. Let E be a metric space and $f: E \rightarrow E$.
The map \tilde{G}_A is said to be topologically semi-conjugate to f and
 $h: \Omega_A \rightarrow E$ is said to be a topological conjugacy, if
 h is continuous and surjective and

$$h(\tilde{G}_A(w)) = f(h(w)), \text{ for any } w \in \Omega_A.$$

$$\text{We denote this fact as } h \circ \tilde{G}_A = f \circ h.$$

You already know that transitivity of matrix A implies topologically mixing of \tilde{G}_A .

Lemma 1. Let \tilde{G}_A be topologically semi-conjugate to f and h be a conjugacy. If \tilde{G}_A is topologically mixing then so is f .

Proof. What does topological mixing of \tilde{G}_A mean?

for any two nonempty open sets $U, V \subset \Omega_A$
there exists n_0 such that

$$(\tilde{G}_A)^n(U) \cap V \neq \emptyset \text{ for every } n > n_0.$$

Assume the contrary.

$$\Rightarrow \exists (\text{open } \hat{U}, \hat{V} \subset E) \nexists (n_0 \in \mathbb{N}) \exists (n > n_0) [f^n(\hat{U}) \cap \hat{V} = \emptyset]$$

By surjectivity: $\exists u_0, v_0 : h(u_0) \subset \hat{U}, h(v_0) \subset \hat{V}$.

By continuity: $\exists U, V : h(\bar{U}) \subset \hat{U}, h(\bar{V}) \subset \hat{V}$.

$$f^n(h(\bar{U})) \cap h(\bar{V}) = \emptyset$$

$$f^n(h(\bar{U})) = f^{n-1}(f(h(\bar{U}))) = f^{n-1}(h(\tilde{G}_A(\bar{U}))) = f^{n-2}h\tilde{G}_A^2(\bar{U})$$

$$h((\tilde{\sigma}_A)^n(\bar{U})) \cap h(V) = \emptyset$$

$$h(A) \cap h(B) \supset h(A \cap B)$$

$$h((\tilde{\sigma}_A)^{n_k}(\bar{U}) \cap V) = \emptyset, \text{ where } n_k \rightarrow \infty \text{ as } k \rightarrow \infty$$

(3)

You already know that topologically mixing map on a space with more than one point has sensitive dependence on initial conditions.

Therefore, in the settings of Lemma 1, if A is transitive, then f has sensitive dependence on initial conditions.