INTEGRAL MODELS OF CUBIC SURFACES

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We fix an arbitrary discrete valuation ring $\mathcal{O}$, with fraction field K, parameter t, and residue field k. My favourites are the local ring $\mathcal{O}_{C,p}$ of a smooth complex curve C at a point $p \in C$, $\mathbb{C}\{t\}$, $\mathbb{Q}[t]$, $\mathbb{F}_p[t]$, $\mathbb{F}_p[[t]]$, the formal completion $\mathbb{Z}_p$ of the integers at p. We will assume that $\operatorname{ch} k \neq 2, 3$. $\overline{k}$ is the *separable* algebraic closure of k, $\overline{K} \supset K$ an unramified extension of K inducing $\overline{k}$, $\overline{\mathcal{O}}$ the integral closure of $\mathcal{O}$ in $\overline{K}$. $\overline{\mathcal{O}}$ is a discrete valuation ring with parameter t. We denote $\Delta = \operatorname{Spec} \mathcal{O}$, $\eta = \operatorname{Spec} K$, $0 = \operatorname{Spec} k$. In what follows all varieties, schemes, morphisms, etc. are tacitly assumed to be defined over Δ, unless otherwise indicated. For a scheme Z, Z_η, Z_0 are its generic and special fiber. A *birational map* is always assumed to be biregular when restricted to the generic fibers. $\overline{\Delta} = \operatorname{Spec} \overline{\mathcal{O}}$, and we denote $\overline{Z}$ the base change to $\overline{\Delta}$, $\overline{Z}_\eta$, $\overline{Z}_0$ its generic and special fiber.

Given a smooth cubic surface $X_K \subset \mathbb{P}^3_K$, we wish to construct a nice integral model for X_K. In other words, we seek a nice $X \subset \mathbb{P}^3_{\mathcal{O}}$ with generic fiber X_K.

1 Definition. a 3-dimensional scheme X over Δ has *cDV* (compound Du Val) singularities if, for every singular $\overline{k}$-rational point $p \in \overline{X}$, there is a surface $\overline{B} \ni p$ with Du Val singularities (i.e., rational double points).

2 Definition. Let $X_K \subset \mathbb{P}^3_K$ be a smooth cubic surface. A subscheme $X \subset \mathbb{P}^3$, flat over Δ, is said to be a standard integral model for X_K if $X_\eta = X_K$ and:

2.1 X has isolated *cDV* singularities,

2.2 X_0 is (reduced and) k-irreducible.

In other words, X has Gorenstein terminal singularities and, if $\operatorname{Pic} X_K = \mathbb{Z}$, $X \to \Delta$ is a Mori fiber space.

I shall soon describe the flowchart of a program to construct standard models. In order to do so, I must first discuss elementary transformations of projective space, and their effect on X.

3 Elementary transformations of projective space. Let $\mathbb{P} = \mathbb{P}^n_{\mathcal{O}}$ be n-dimensional projective space over Δ, $L = L_d \subset \mathbb{P}_0$ be a d-dimensional linear subspace, defined over k. If $d \leq n-1$, there is a birational transformation $\Phi = \Phi_L : \mathbb{P} \dashrightarrow \mathbb{P}$, centered at L. This is nothing but projection from L, and, in homogeneous coordinates, $\Phi : (x_0 : \cdots : x_n) \to (tx_0 : \cdots : tx_d : x_{d_1} : \cdots : x_n)$. Φ fits into a commutative diagram:

$$\begin{tikzcd} & A \arrow[dl, "f"'] \arrow[dr, "g"] & \\ \mathbb{P} \arrow[rr, dashed, "\Phi"] & & \mathbb{P}^+ \end{tikzcd}$$

where $f = bl_L\mathbb{P} : A = Bl_L\mathbb{P} \to \mathbb{P}$ is the blow up of $L \subset \mathbb{P}$. We denote F, $G \subset A$ the f and g-exceptional divisors, so that $f(G) = \mathbb{P}_0$ and $g(F) = \mathbb{P}^+_0$. Clearly:

$$K_A = f^*(K_{\mathbb{P}}) + (n-d)F \tag{3.1}$$
$$= g^*(K_{\mathbb{P}^+}) + (d+1)G \tag{3.2}$$

I will often simply denote K, K^+ the canonical classes of $\mathbb{P}, \mathbb{P}^+$.

Let now $X \subset \mathbb{P} = \mathbb{P}^3_\Delta$ be a flat subscheme, with $X_\eta \subset \mathbb{P}_\eta$ a smooth cubic surface. Then $X_0 \subset \mathbb{P}_0$ is the scheme of zeros of a homogeneous cubic form. In the following three lemmas we discuss, under different circumstances, the effect, on X, of elementary transformations of $\mathbb{P}$. The resulting modifications of X will be used to improve the singularities of X until a distinguished integral model of X_η is reached.

For a variety V with canonical singularities, $e(V)$ denotes the number of crepant valuations. $n(X)$ denotes the number of k-irreducibe components of the central fiber X_0.

4 Lemma. *(blowing up the plane) Assume X_0 contains a 2-plane $L \subset \mathbb{P}_0$, defined over k. Let $1 \leq \mu \leq 3$ be the generic multiplicity of X along L. Let $\Phi = \Phi_L : \mathbb{P} \dashrightarrow \mathbb{P}^+$, $X^+ = \Phi_* X$:*

4.1 Let ν be a valuation, with small center on $\mathbb{P}^+$, and discrepancy $a(\nu, K^+ + X^+) \leq 0$. Then:

$$a\big(\nu, K^+ + X^+\big) \geq a\big(\nu, K + X + (\mu - 1)\mathbb{P}_0\big)$$

4.2 If X has canonical singularities, so does X^+, and $e(X^+) < e(X)$, or $e(X^+) = e(X)$ and $n(X^+) < n(X)$.

Proof. Let $Z = f_*^{-1} X \subset A$. First of all let us prove that:

$$g^* X^+ = Z + (3 - \mu)G$$

Indeed, in a neighborhood of the generic point of L, X is given by an equation $x^k+t\varphi(t,x)=0$. The relevant chart for the blow up is $t=xt'$, so $x^k+t\varphi(t,x)=x^k+xt'\varphi(xt',x)=x^\mu\big(x^{k-\mu}+t'\varphi'(t',x)\big)$. Then $g^*X^+=Z+\big((3-k)+(k-\mu)\big)G=Z+(3-\mu)G$.

The crucial formulas are:

$$\begin{aligned} f^*\big(K+X\big) &= K_A+Z+(\mu-1)F\\ g^*\big(K^++X^+\big) &= K_A+Z-\mu G\end{aligned}$$

Then:

$$\begin{aligned} a(\nu,K^++X^+)=a(\nu,K_A+Z-\mu G)\geq a(\nu,K_A+Z-(\mu-1)G) &=\\ =a(\nu,K+X-(\mu-1)\mathbb{P}_0)&\end{aligned}$$

which proves 4.1. For the proof of 4.2, the important observation is that:

$$\begin{aligned} g^*K_{X^+} &= K_Z-a^+G|_Z\\ f^*K_X &= K_Z+aF|_Z\end{aligned}$$

with $a^+>0$ and $a\geq 0$. Let ν be a valuation, with small center on X^+, and discrepancy $a(\nu,K_{X^+})\leq 0$. Since $a^+>0$, $\nu\neq\nu_{G|Z}$, so ν has small center in Z also. Then:

$$0\geq a(\nu,K_{X^+})=a(\nu,K_Z-a^+G|_Z)\geq a(\nu,K_Z+aF|_Z)=a(\nu,K_X)\geq 0,$$

which means that all inequalities are equalities. First of all, this says that X^+ has canonical singularities, and more than that, every valuation which contributes to $e(X^+)$ also contributes to $e(X)$, so $e(X^+)\leq e(X)$. Also, since $a^+>0$, $C_Z(\nu)\not\subset G|_Z$. But then $C_Z(\nu)\subset F|_Z$ and $a=0$. If $F|_Z$ is a divisor, we have a valuation, namely $\nu_{F|Z}$, which contributes to $e(X)$ but not to $e(X^+)$, which means that $e(X^+)<e(X)$. Otherwise, $Z\to X$ is small, but clearly $Z\to X^+$ is not small, so $n(X^+)<n(X)$. This case does actually happen (obviously we must have $\mu=1$). $\square$

5. Lemma. *(blowing up the line) Assume X has generic multilicity $2\leq\mu\leq 3$ along a line $L\subset\mathbb{P}_0$, defined over k. Let $\Phi=\Phi_L:\mathbb{P}\dashrightarrow\mathbb{P}^+$, $X^+=\Phi_*X$:*

5.1 For ν as in 4.1:

$$a\big(\nu,K^++X^+\big)\geq a\big(\nu,K+X+(\mu-2)\mathbb{P}_0\big)$$

5.2 If X has canonical singularities, so does X^+, and $e(X^+) < e(X)$.

Proof. The proof is very similar to the proof of 4. Let $Z = f_*^{-1}X$. If $L^+ = g(G)$, it is quite clear that $\mu^+ = 3 - \mu$, and:

$$f^*\big(K + X\big) = K_A + Z + (\mu - 2)F$$
$$g^*\big(K^+ + X^+\big) = K_A + Z - (\mu - 1)G$$

Then:

$$a(\nu, K^+ + X^+) = a(\nu, K_A + Z - (\mu - 1)G) \geq a(\nu, K_A + Z - (\mu - 2)G) = \\ = a(\nu, K + X - (\mu - 2)\mathbb{P}_0)$$

which proves 5.1. The proof of 5.2 is, word by word, the same as the proof of 4.2, with the difference that now, since $\mu \geq 2$ by assumption, $Z \to X$ can never be small, which explains the stronger conclusion. $\square$

6. Lemma. *(blowing up the point) Let $p \in X$ be a k-rational point of multiplicity $\mu = 3$. Let $\Phi = \Phi_{\{p\}} : \mathbb{P} \dashrightarrow \mathbb{P}^+$, $X^+ = \Phi_* X$:*

6.1 For ν as in 4.1 or 5.1:

$$a\big(\nu, K^+ + X^+\big) \geq a\big(\nu, K + X\big)$$

6.2 Same as 5.2.

Proof. Let $Z = f_*^{-1}X$. If $L^+ = g(G)$, it is quite clear that $\mu^+ = 3 - \mu = 0$, and:

$$f^*\big(K + X\big) = K_A + Z$$
$$g^*\big(K^+ + X^+\big) = K_A + Z - G$$

Then:

$$a(\nu, K^+ + X^+) = a(\nu, K_A + Z - G) \geq a(\nu, K_A + Z) = a(\nu, K + X)$$

which proves 6.1. The proof of 6.2 is, word by word, the same as the proof of 5.2. $\square$

We are now ready to describe the (still conjectural) procedure to construct distinguished integral models of cubic surfaces. Fix a smooth cubic surface $X_K \subset \mathbb{P}^3_K$.

7 Flowchart.

7.0 Let $X \subset \mathbb{P} = \mathbb{P}^3_{\mathcal{O}}$ be an arbitrary flat closure of X_K.

7.1 Does X have generic multiplicity $\mu \geq 2$ along a 2-plane $L \subset \mathbb{P}_0$? If yes, letting $\Phi : X \dashrightarrow X^+$ be as in 4, go back to 7.1 with X^+ in place of X. If not, go to 7.2.

7.2 Does X have generic multiplicity $\mu = 3$ along a line $L \subset \mathbb{P}_0$? If yes, letting $\Phi : X \dashrightarrow X^+$ be as in 5, go back to 7.1 with X^+ in place of X. If not, go to 7.3.

7.3 Does X_0 contain a plane? If yes, letting $\Phi : X \dashrightarrow X^+$ be as in 4, go back to 7.1 with X^+ in place of X. If not, go to 7.4.

7.4 Does X have generic multiplicity $\mu = 2$ along a line $L \subset \mathbb{P}_0$? If yes, letting $\Phi : X \dashrightarrow X^+$ be as in 5, go back to 7.1 with X^+ in place of X. If not, go to 7.5.

7.5 Is there a k-rational point $p \in X$ of multiplicity 3? If yes, letting $\Phi : X \dashrightarrow X^+$ be as in 6, go back to 7.1 with X^+ in place of X. If not, one of the following is true:

7.5.1 X is a standard model.

7.5.2 X is the *exceptional model* described below. The exceptional model is birational to a *special index 2 model*.

We need to describe the exceptional model, prove the statement in 7.5, and show that the program terminates. This last part, unfortunately, is still conjectural:

8 Conjecture. *The program terminates.*

Lemmas 4–6 show that, in some sense, each of the steps in the program improves singularities. Perhaps there is an invariant $\delta X \in \mathcal{O}$, such that $\delta X^+ < \delta X$, presumably related to the scheme parametrising lines in X.

9 Description of the exceptional model. X is an *exceptional model* if the following 3 conditions hold:

9.1 $\overline{X}_0 = L_1 + L_2 + L_3$ is union of 3 planes, none of which is defined over k.

9.2 X is singular along $C = (L_1 \cap L_2) + (L_1 \cap L_3) + (L_2 \cap L_3)$.

9.3 Let $p \in X_0$ be the triple point. Then X has multilicity $\mu = 2$ at p.

The conditions imply that X has cA_1 singularities and C is the A_1 curve. A model with terminal singularities can be obtained by blowing up C and contracting the strict ransforms of the L_is. The resulting variety is a *special index 2 model*, and it has three index 2 geometric closed points, permuted by the Galois group $Gal(\overline{k}/k)$.

10 Remark. If:

$$X = \Big(\sum_{k\geq 0} t^k F_k = 0\Big)$$

with $F_k = F_k(x_0, \dots, x_3)$ homogeneous of degree 3, 9.2 means that $C \subset (F_1 = 0)$, 9.3 means that $p \notin (F_2 = 0)$.

11 Theorem. *Let $X \subset \mathbb{P}^3_{\mathcal{O}}$ be a subscheme, flat over $\mathcal{O}$, whose generic fiber $X_\eta \subset \mathbb{P}^3_\eta$ is a smooth cubic surface. Assume the following:*

11.1 X_0 is k-irreducible. This is equivalent to saying that X_0 contains no 2-plane defined over k, and it implies that X_0 is reduced. In particular, X is nonsingular in codimension one, hence normal.

11.2 X is nonsingular at the generic point of every line $L \subset \mathbb{P}_0$ defined over k.

11.3 X has multiplicity $\mu \leq 2$ at every k-rational point $p \in X$.

Then either X is a standard model, or X is an exceptional model.

Proof. X_0 is reduced and k-irreducible, so $\overline{X}_0$ is reduced. If X_0 is not geometrically irreducible, $\overline{X}_0 = L_1 + L_2 + L_3$ is the union of three 2-planes L_i, none of wich is defined over k: indeed, if $\overline{X}_0 = L + Q$, where L is a 2-plane and Q an irreducible quadric, the 2-plane L is necessarily defined over k.

I will prove that, unless X is an exceptional model, X has isolated singularities. The result will then follow from the next lemma 12.

If X_0 is geometrically irreducible, the nonnormal locus of X_0, if nonempty, consists of a line. Then X has isolated singularities by condition 11.2.

Otherwise, $\overline{X}_0 = L_1 + L_2 + L_3$ as above. Let $C = (L_1 \cap L_2) + (L_1 \cap L_3) + (L_2 \cap L_3)$. No component of C is defined over k, for otherwise one of the L_i's is defined over k: for instance if $L_1 \cap L_2$ is defined over k, L_3 is also necessarily defined over k. This means that either X has isolated singularities, or the singular locus of X is all of C. But in this case, since $p = L_1 \cap L_2 \cap L_3$ is necessarily k-rational, X is an exceptional model. $\square$

12 Lemma. *Let X be as in 11. Assume:*

12.1 X_0 is k-irreducible.

12.2 X has isolated singularities.

12.3 X has multiplicity $\mu \leq 2$ at every k-rational point $p \in X$.

Then X has cDV singularities, i.e., X is a standard model.

The proof of 12 is based on the following elementary lemma and its corollary:

13 Lemma. *Let A be affine 3-space with coordinates x, y, z, $(p \in B) = \big(0 \in (f(x,y,z) = 0)\big) \subset A$ be the germ at the origin of a normal singularity. Write $f = \sum f_k$ with f_k homogeneous of degree k. Then:*

13.1 If $(f_2 = 0)$ is a reduced conic, $0 \in B$ is a Du Val singularity of type A_n, for some n.

13.2 If $f_2(x,y,z) = x^2$ and $f_3(0,y,z)$ has 2 distinct roots, $0 \in B$ is a Du Val singularity of type D_n, for some n.

13.3 If $f_2(x,y,z) = x^2$, $f_3(0,y,z) = y^3$, $xz^2 \in f_3$ and $z^4 \notin f_4$, $0 \in B$ is a Du Val singularity of type E_6.

Proof. This is all well known and easy. As an example, I will outline the proof of 13.3, which is the hardest. Write:

$$f(x,y,z) = x^2 + y^3 + xg_2(y,z) + h(x,y,z)$$

with g_2 homogeneous of degree 2, $z^2 \in g_2$,$h = O(4)$, $z^4 \notin h$. Then:

$$f = \left(x + \frac{g_2}{2}\right)^2 + y^3 - \frac{g_2^2}{4} + h$$

Changing coordinates $x \to x + \frac{g_2}{2}$, f is transformed to

$$f' = x^2 + y^3 - \frac{g_2^2}{4} + h'$$

where $h' = O(4)$ and, most importantly, $z^4 \notin h'$. Since $z^2 \in g_2$, this implies that $z^4 \in f'$. Now the vertices x^2, y^3, z^4 generate a face F of the Newton polyhedron for f'. The Jacobian ideal of $x^2+y^3+z^4$ is $J = (2x, 3y^2, 4z^3)$ and, if $\operatorname{ch} k \neq 2,3$, $\mathcal{O}_{A,0}/J$ has a basis represented by monomials lying entirely below F. Using [AVGZ], vol. 1, 12.6, theorem on page 174, this implies that f' is formally equivalent to $x^2+y^3+z^4$. So $0 \in B$ is formally equivalent to an E_6 singularity, hence it is an E_6 singularity. □

14 Corollary. *Let A be affine 4-space with coordinates x, y, z, t, $p \in X = 0 \in (f(x,y,z,t) = 0) \subset A$ be the germ at the origin of an isolated singularity. Then:*

14.1 If $(f_2 = 0)$ is a reduced quadric, $p \in X$ is a cA_n singularity, for some n.

14.2 If $f_2 = x^2$ and $f_3(0,y,z,t) = 0$ is not a triple plane, $p \in X$ is a cD_n singularity, for some n. □

15. Proof of 12. Let $p \in \overline{X}$ be a singular point. Let $p \in \overline{B}$ be a hyperplane section of $\overline{X}$, general among those passing through p. We will prove that $p \in \overline{B}$ is a Du Val singularity. Let $\overline{B}_0 = B \cdot \overline{X}_0$, then $\overline{B}_0$ is a reduced plane cubic, and $p \in \overline{B}_0$ is a singular point. We distinguish five cases:

15.1 $\overline{B}_0$ is a nodal rational curve and $p \in \overline{B}_0$ the node. Then $p \in \overline{X}$ is cA_n by 14.1.

15.2 $\overline{B}_0$ is a cuspidal rational curve and $p \in \overline{B}_0$ the cusp. This is the hardest case and is treated below.

15.3 $\overline{B}_0 = L + C$ where L is a line and C a reduced conic. This case does not occur because in this case X_0 must contain a 2-plane defined over k, contradicting the assumptions.

15.4 $\overline{B}_0 = L_1 + L_2 + L_3$ and $p \in \overline{B}_0$ is a double point. Then $p \in \overline{X}$ is cA_n by 14.1.

15.5 $\overline{B}_0 = L_1 + L_2 + L_3$ and $p \in \overline{B}_0$ is the triple point. This case will be discussed momentarily.

I will now discuss cases 15.5, 15.2, in this order.

In case 15.5, after base change to $\overline{\mathcal{O}}$, in suitable affine coordinates near p, $\overline{B}_0$ is described by an equation of the form:

$$f = xy(x+y) + \sum_k t^k f_k(x,y) = 0$$

where f_k is a (not necessarily homogeneous) polynomial of degree ≤ 3 in x, y. By assumption, the origin is a point of multiplicity $\mu \leq 2$. This means that either f_1 contains a linear term, or f_2 contains a constant term. In the second case, $p \in \overline{B}$ is Du Val by 13.1–2. In the first case, $p \in \overline{B}$ is A_n by 13.1. This completes the proof in case 15.5.

So from now on we assume to be in case 15.2. We divide the proof in two parts, according to wether X_0 is normal or not.

15.2.1 X_0 is normal. It is well known that a normal Del Pezzo surface, which is not the cone over a smooth elliptic curve, has Du Val singularities. In the case at hand, $\overline{X}_0$ is clearly not an elliptic cone (otherwise we would be in case 15.5), so it must have Du Val singularities, and $\overline{X}$ is then cDV. For sake of completeness, and since the proof is easy anyway, I will provide the argument for cubics. Since $\operatorname{ch} k \neq 2$, the cusp $p \in \overline{B}_0$ is a standard cusp and, after base change to $\overline{\mathcal{O}}$, in suitable affine coordinates near p, $\overline{X}_0$ is described by an equation of the form:

$$x^2 + y^3 + zg(x,y,z) = 0$$

where g is a polynomial of degree ≤ 2 in x, y, z. Here $\overline{B}_0 = (z = 0)$ is a general hyperplane section containing the origin, so g does not contain any constant or linear terms, in other words g is homogeneous of degree 2. If $g(0,y,z) \neq 0$, $p \in \overline{B}_0$ is of type D_n by 13.2. If $g(0,y,z) = 0$ and $xz \in g$, $p \in B_0$ is of type E_6 by 13.3. So we may assume that $g(0,y,z) = 0$ and $xz \notin g$. In this case:

$$\overline{X}_0 = (f = x^2 + y^3 + \alpha z x^2 + \beta z x y = 0)$$

and this surface is singular along the line $x = y = 0$, a contradiction.

15.2.2X_0 is not normal. The nonnormal locus of X_0 consists of a line, containing the point p, and X_0 has a simple cusp along this line. Since $\operatorname{ch} k \neq 2$, the cusp is a standard cusp and, after base change to $\overline{\mathcal{O}}$, in suitable affine coordinates near p, $\overline{X}_0$ is described by an equation of the form:

$$x^2 + y^3 + zg_2(x,y) = 0$$

where g_2 is homogeneous of degree 2 (the nonnormal line is the line $x = y = 0$). If $g_2 \neq 0$, $p \in \overline{X}_0$ is of type D_n by 13.2. Otherwise $g_2 = 0$, which means that X_0 is the cone over a cuspidal rational curve and $p \in \overline{X}_0$ is not the vertex, but lies on the cuspidal line. Now $\overline{X}$ itself is described by an equation:

$$f = x^2 + y^3 + tf_1(x,y,z) + t^2 f_2(x,y,z) + t^3 f_3(x,y,z) + \cdots = 0$$

where f_k is a polynomial of degree ≤ 3. The crucial piece of information here is that X has *isolated singularities*, in particular X is nonsingular at the generic point of $t = x = y = 0$. This means that $z^k \in f_1$ for some k, and necessarily $k \leq 3$. We will see that this implies that X has cDV singularities.

15.2.2.1 If f_1 contains a linear term, or f_2 a constant term, $p \in \overline{X}$ is cA_n by 14.1. From now on assume that this is not the case.

15.2.2.2 If, then, $f_1(0,y,z)$ contains a quadratic term, or $f_2(0,y,z)$ a linear term, or $f_3(0,y,z)$ a constant term, $p \in \overline{X}$ is cD_n by 14.2. From now on assume that this is not the case.

15.2.2.3 We then have:

$$f = x^2 + y^3 + txg_2(x,y,z) + th_3(y,z) + t^2 h(x,y,z,t)$$

where:

a) $g_2(x,y,z)$ is a polynomial of degree ≤ 2 (not necessarily homogeneous), and $g_2(0,y,z)$ is homogeneous of degree 3;

b) $h_3(y,z)$ is a homogeneous polynomial of degree 3 and $z^3 \in h_3$;

c) finally, $h(x,y,z,t)$ is a power series vanishing at the origin, and $h(0,y,z,t)$ contains no linear terms.

Then:

$$f = \left(x + \frac{tg_2}{2}\right)^2 + y^3 + th_3(y,z) + t^2 h'(x,y,z,t)$$

which, via $x \to x + \frac{tg_2}{2}$, transforms to:

$$x^2 + y^3 + th_3(y,z) + t^2 h''(x,y,z,t)$$

Now, as in the proof of 13.3, $tz^3 \in h_3$ implies (if $\operatorname{ch} k \neq 2,3$) that $p \in \overline{X}$ is a cE_6 singularity. $\square$

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