Biomedical Engineering Year 1 **Mathematics Problems 7**

1. Evaluate the following integrals.

(i) $\displaystyle\int \frac{dx}{x\,(2-3x)}$.
(ii) $\displaystyle\int \frac{dx}{x\,(x^2+1)}$.
(iii) $\displaystyle\int \left(a^2-x^2\right)^{1/2}\,dx$.

(iv) $\displaystyle\int \frac{x^{1/2}}{1+x}\,dx$.
(v) $\displaystyle\int \frac{3x^2+4x+1}{x^3+2x^2+x}\,dx$.
(vi) $\displaystyle\int \frac{\tan^{-1}x}{1+x^2}\,dx$.

(vii) $\displaystyle\int \frac{dx}{x\ln x}$.
(viii) $\displaystyle\int \tan^2 x\,dx$.
(ix) $\displaystyle\int \frac{dx}{1+\tan x}$. (Try $u=\tan x$.)

(x) $\displaystyle\int \frac{dx}{2+\sin x}$.
(xi) $\displaystyle\int x^2\cos x\,dx$.
(xii) $\displaystyle\int \mathrm{e}^x\cos x\,dx$.

(xiii) $\displaystyle\int \frac{x\cos x}{\sin^2 x}\,dx$.
(xiv) $\displaystyle\int x^k\ln x\,dx$ ($k\neq -1$).
(xv) $\displaystyle\int_{-1}^{1} \frac{dx}{x^2+2x+5}$.

(xvi) $\displaystyle\int_0^{\pi/2} \frac{dx}{(1+\cos x)^2}$.
(xvii) $\displaystyle\int \frac{dx}{(x^2+4x+13)^{1/2}}$.

2. Let $\displaystyle I_n = \int_0^\infty x^n\,\mathrm{e}^{-x^2}\,dx$, where n is a positive integer. Prove that

$$I_{n+1} = \frac{1}{2}\,n\,I_{n-1} \qquad \text{and hence evaluate} \qquad I_5 = \int_0^\infty x^5\,\mathrm{e}^{-x^2}\,dx\,.$$

3. If $\displaystyle u_n = \int x^n\cos x\,dx$ and $\displaystyle v_n = \int x^n\sin x\,dx$ show that

$$u_n = x^n\sin x - n\,v_{n-1} \quad \text{and} \quad v_n = -\,x^n\cos x + n\,u_{n-1}\,.$$

Hence evaluate $\int x^4\sin x\,dx$.

4. Let $\displaystyle I_n = \int_0^{\pi/4} \tan^n x\,dx$. Prove that $\displaystyle I_n = \frac{1}{n-1} - I_{n-2}$

and hence evaluate I_5 .

Answers for Problems 6

1. $2n\pi + i\cosh^{-1}4$ or $2n\pi \pm i\ln(4+15^{1/2})$.
3. 0 , $\ln 2$.
6. c , -2 .
7. $\mathrm{sech}^2 x$, $-\mathrm{sech}x\,\tanh x$, $2/\sinh 2x$.
9. $\tanh^{-1}x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots,\quad -1<x<1$.
   $\tan^{-1}s = s - \frac{s^3}{3} + \frac{s^5}{5} + \ldots$.
10. $h = \frac{1}{2}\ln 9 \approx 1.10$.

[P.T.O. for some hints.]

Some Hints for Problems 7

1. (i) Use partial fractions.

   (iii) Substitute $x = a\sin\theta$, then use a trigonometric identity.

   (iv) Substitute for the "difficult" term, i.e. put $u = x^{1/2}$.

   (xi) Use integration by parts, twice, to remove the x^2 term.

   (xv) Complete the square.

   (xvii) Complete the square, substitute $x + 2 = 3\tan\theta$. $\int \frac{d\theta}{\cos\theta} = \ln|\sec\theta + \tan\theta| + c$.

2. In $I_{n+1} = \int_0^\infty x^{n+1}\,\mathrm{e}^{-x^2}\,dx$, write $x^{n+1}\,\mathrm{e}^{-x^2} = (x^n)\,(x\mathrm{e}^{-x^2})$ and use integration by parts.