

1. Find the first order ODEs for which the following functions are the general solutions, by differentiating in order to eliminate the arbitrary constant c .

$$(i) \quad y = (c + x) \sin x. \quad (ii) \quad x = c \exp\left(-\frac{x}{y}\right).$$

2. Solve the following first order ODEs. Find the general solution when no condition is given.

$$(i) \quad \frac{dy}{dx} = \frac{y+1}{x+1}, \quad y(0) = 1. \quad (ii) \quad (1+x^2) \frac{dy}{dx} + 4xy = 0, \quad y(0) = \frac{1}{2}.$$

$$(iii) \quad (1-x^2) \frac{dy}{dx} + xy = x, \quad y(0) = 2. \quad (iv) \quad x^2 \frac{dy}{dx} + xy - y^2 = 0.$$

$$(v) \quad 2xy \frac{dy}{dx} + 3x^2 + y^2 = 0. \quad (vi) \quad (x^4 - 2xy^3) \frac{dy}{dx} = 2x^3y - y^4.$$

$$(vii) \quad \frac{dy}{dx} + 2(\tan x)y = \sin x, \quad y(\pi) = -3. \quad (viii) \quad \frac{dy}{dx} + 2xy = 2e^{-x^2}.$$

$$(ix) \quad x \frac{dy}{dx} + 2y = x^2 - x + 1, \quad y(1) = \frac{1}{3}.$$

3. Show that the ODE $\frac{dy}{dx} = \frac{2x + 2y - 2}{3x + y - 5}$

can be made homogeneous by changing the variables to X and Y , where $x = X + A$ and $y = Y + B$, and choosing a suitable constants A and B . Hence find $y(x)$.

4. Show that the following ODE's are exact and hence solve them.

$$(i) \quad (9x^2 + y - 1) dx + (x - 4y) dy = 0. \quad (ii) \quad (xy^2 + y) dx + (x^2y + x) dy = 0.$$

5. Multiply the ODE $(5x^2 + 12xy - 3y^2) dx + (3x^2 - 2xy) dy = 0$ by x^n , determine n so that the new equation is exact, and hence solve the equation.

6. Solve the equation $(3xy^3 - 4xy + y) \frac{dy}{dx} + y^4 - 2y^2 = 0$

by finding an integrating factor which is a function of xy^2 , i.e. multiply by $f(z)$ where $z = xy^2$, and choose f so that the resulting equation is exact. [$f'(z)(z+1) = f(z)$.]

7. A circular funnel of height H and base radius R is placed with its axis vertical and vertex down. Initially the funnel is filled with liquid which then escapes through a small hole of radius a at the vertex. The velocity of the escaping fluid is $ky^{1/2}$, where k is a constant and y is depth of fluid remaining in the funnel. Find the time taken for the funnel to empty.

Answers for Problems 8

1. (i) $2(3)^{1/2}$. (ii) 3. (iii) -1 . (iv) and (v) The integrals diverge.
 2. (i) 0, (ii) $\frac{2}{5}(1 - e^{-15})$. 3. $\sinh 1$. 4. $2 + \frac{1}{2} \ln 3$. 6. 4.