

M4P14 Elementary Number Theory Enhanced Coursework

Alessio Corti

15th December 2008

This enhanced coursework is due by **Wednesday, 28th Jan 2009**. Late submissions cause problems with the whole system so I will happily penalise anyone who hands in anything after this time. Early submission is hence strongly encouraged. Remember, files get viruses, printers break, and so on and so on, especially when there are many people all trying to use the same computers and all working to the same deadlines. Don't get caught out.

1 Titles

You will write a short essay on one of the following two topics:

1. Continued Fractions and the Pell equation
2. Reduction theory for positive integer quadratic forms (and the class number of imaginary quadratic fields)

2 Suggestions and references

2.1 First Essay

If you choose the first title, then you should summarise the theory of continued fractions only to the extent that is required for you to discuss the applications to the Pell equation. You should also make sure that you include a handful of worked out numerical examples of solving particular Pell equations.

Here are three references (you should probably look at all three but possibly make your own research in the library):

- 1 A. Baker *A concise introduction to the theory of numbers*, Cambridge University Press; Ch 6 and §8.1.
- 2 G.H. Hardy and E.M. Wright, *An introduction to the theory of numbers*, Oxford Science Publications; Ch X and XIV.
- 3 J. Silverman, *A friendly introduction to number theory*, Prentice Hall; Ch 30, 31, 32.

2.2 Second Essay

If you choose to do the second essay, then you should discuss Lagrange's theory of reduction of positive definite integer quadratic forms following for example Baker. If you know something about algebraic number theory, it would be nice to discuss the 1-1 correspondence between positive quadratic forms with discriminant $\Delta < 0$ and ideals in the ideal class group of the ring of integers in the imaginary quadratic extension $\mathbb{Q}(\sqrt{\Delta})$ as explained, for example, in the book of Swinnerton-Dyer. This complements beautifully some of the results in the course and builds a bridge with the algebraic number theory course.

Here are two references:

- 1 A. Baker *A concise introduction to the theory of numbers*, Cambridge University Press; Ch 6 and §8.1.
- 2 P. Swinnerton-Dyer, *A Brief Guide to Algebraic Number Theory*, London Mathematical Society Student Texts **50**, Cambridge University Press 2001.

3 What am I looking for?

Firstly: You are not being asked to do original work here (although of course you can write up numerical experiments that you do, if you decide to approach the project from a computational point of view). On the other hand, you are also not just expected to copy things straight out of books or off the internet. Every year someone finds some book or web page they hope the markers won't know about and copies huge chunks out of it, essentially word for word, and every year these people get 0 marks. A related point: beware the internet. There's lots of good stuff out there, but there's also lots of junk.

Understand how much work you are supposed to be doing: If you hand in 5 pages, your project will be too short. Between 10 and 20 typed pages is OK, or perhaps more than 10 if you are writing by hand. Handwritten projects are *fine*, as are ones written on a computer.

Again, do **not** just copy out of a reference. What I am looking for is some *imagination!* For example, skip some proof, but fill in the details of some others if you find them hard to understand. If Silverman (or some other reference) says "We proved this in Chapter 20", then consider proving it in your essay. Rewrite the proofs, re-order material if you feel it's more appropriate. **Work out some examples.** The key thing is to somehow *show that you understand what you're writing*. Try reading round the subject! There are libraries on IC campus! Trying to explain your own point of view of the subject, or including things not mentioned in the suggested references—you might think these things are risky, but they will be worth more marks than just copying proofs out of a book.