

## TCC - Problems Sheet 1

1. Show that if  $\{f_n\} \subset \mathcal{H}$  is weakly convergent, then  $\{\|f_n\|\}$  is bounded.
2. Show that the unit ball in  $\mathcal{H}$  is weakly sequentially compact, that is, if  $\{\|f_n\|\}$  is bounded, then there is a subsequence  $\{f_{n_k}\}$  which converges weakly to some  $f \in \mathcal{H}$ .
3. Show that  $\|T\| = \|T^*\|$ .
4. Show that if an operator is self-adjoint then it is closed.
5. Go through the proof of Lemma 2.4 in Basics.
6. Prove that the multiplication operator  $T_\varphi$  is densely defined and is bounded iff  $\varphi$  is essentially bounded.