BSc and MSci EXAMINATIONS (MATHEMATICS)

January 2009

M1GLA (Test)

Geometry and Linear Algebra

- Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers.
- The question in Section A will be worth $1\frac{1}{2}$ times as many marks as either question in Section B.
- Calculators may not be used.

SECTION A

1. (i) Find all values of $a \in \mathbb{R}$ such that the line $\{(3,1) + \lambda(-1,a) | \lambda \in \mathbb{R}\}$ is parallel to the line with equation $2x + 3y = 1$.

   (ii) Let $\Pi \subset \mathbb{R}^3$ be the plane perpendicular to the line $\{(1,0,-1) + \lambda(1,2,-3) | \lambda \in \mathbb{R}\}$, such that $(1,1,1) \in \Pi$. Find the perpendicular distance from the point $(0,1,-1)$ to Π.

   (iii) Find the coordinates of the point $X \in \mathbb{R}^2$ such that $|AX| = |BX| = |CX|$, where $A = (0,0)$, $B = (1,3)$, $C = (-1,5)$.

   (iv) Find the foci of the ellipse $\frac{x_1^2}{25} + \frac{x_2^2}{16} = 1$.

   (v) Find the type of the conic $-7x_1^2 + 48x_1x_2 + 7x_2^2 = c$ for all values of $c \in \mathbb{R}$.

   (vi) Find all 3×3 matrices A satisfying the condition $A^T = -A$ such that $I + A$ has no inverse.

SECTION B

2. (i) Define what is meant by an echelon form of a matrix.

   (ii) Define the elementary row operations.

   (iii) Briefly explain why every matrix can be reduced to echelon form by elementary row operations.

   (iv) Find the inverse of the matrix

$$\begin{pmatrix} 8 & 4 & 2 & 1 \\ 4 & 2 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

   by reducing it to the identity matrix by row operations.

3. (i) Define what is meant by an orthogonal matrix.

   (ii) Let A be a 2×2 orthogonal matrix such that $\det A = -1$. What can you say about the eigenvalues of A?

   (iii) Find all values of $a \in \mathbb{R}$ such that the following system of linear equations has infinitely many solutions

$$\begin{aligned} ax_1 + x_2 + x_3 &= 1 \\ x_1 + ax_2 + x_3 &= 1 \\ x_1 + x_2 + ax_3 &= -2 \end{aligned}$$