M1GLA Geometry and Linear Algebra

Exercise Sheet 2

(*not for assessment*)

1. Let a, b be the points $(1, 2), (-2, 5)$. Find

(i) in vector form, the line L through a and b

(ii) a vector perpendicular to L

(iii) a scalar α such that $(\alpha, 6)$ lies on L

(iv) the point of intersection of L with the line $\{(1, 1) + \lambda(1, 2) : \lambda \in \mathbb{R}\}$.

2. Let a, b, c be the three points $(1, 2)$, $(-1, 3)$, $(0, 7)$. Showing the steps of your calculations, find

(i) the equation of the line L_1 through a and b (in the form $px_1 + qx_2 + r = 0$)

(ii) the equation of the line L_2 through c parallel to L_1

(iii) the equation of the line L_3 through c perpendicular to L_1

(iv) the point of intersection of L_1 and L_3

(v) the perpendicular distance between c and L_1

(vi) the angle between L_1 and the line through a and c.

3. Let $\{a, b, c\}$ be a triangle. A line passing through one of the corners of the triangle and perpendicular to the opposite side is called an *altitude* of the triangle.

Show that the equation of the altitude through the corner a is

$$x.(b - c) = a.(b - c).$$

Prove that the three altitudes of a triangle meet in a common point.

Find the point where the altitudes meet in the case where a, b, c are $(1, 2)$, $(2, -1)$, $(0, 3)$.

4. Prove Proposition 2.3 of the lectures: any two non-parallel lines meet in a unique point.

5. Prove that if two lines $px_1 + qx_2 + r = 0$ and $p'x_1 + q'x_2 + r' = 0$ are perpendicular, then $pp' + qq' = 0$.