## M1GLA Geometry and Linear Algebra Exercise Sheet 4

1. Reduce the following conics to standard form by rotation and translation, and make rough sketches:

- (i)  $x_1^2 2\sqrt{3}x_1x_2 + 3x_2^2 + 2\sqrt{3}x_1 + 2x_2 = 0$ (ii)  $x_1^2 - 4x_1x_2 + 4x_2^2 + 6x_1 - 12x_2 + 9 = 0$ .
- **2.** Find all values of c for which the equation  $x_1^2 + 2cx_1x_2 + x_2^2 1 = 0$  defines
  - (i) a hyperbola
  - (ii) an ellipse
  - (iii) neither of these.

What is the conic in case (iii) ?

**3.** For what values of e does the focus-directrix equation  $||x - p|| = e \cdot \operatorname{dist}(x, L)$  define a circle  $(z_1^2 + z_2^2 = a^2)$  or a right hyperbola  $(z_1^2 - z_2^2 = a^2)$ ?

## 4. Consider the equation

$$(x_1+r)^2 + \frac{x_2^2}{1-e^2} = \frac{s^2e^2}{(1-e^2)^2}$$

where  $r = se^2/(1-e^2)$ , obtained from the focus-directrix definition of conics in lectures. Suppose e < 1, so the equation defines an ellipse.

(a) Comparing the equation to the standard equation  $y_1^2/a^2 + y_2^2/b^2 = 1$  for an ellipse, where  $y_1 = x_1 + r$ ,  $y_2 = x_2$ , find expressions for e, s and r in terms of a and b, and vice versa. (Take a, b > 0.)

(b) Show that in the y-system the focus is p = (ae, 0) and the directrix has equation  $y_1 = a/e$ .

(c) Show that for the same ellipse, the point p' = (-ae, 0) (in the y-system) and the line L' with equation  $y_1 = -a/e$ , also have the focus-directrix property.

(d) Hence show that for any point x on the ellipse, the sum of the distances from x to p and p' is equal to 2a.

5. The Republic of Transdniestria has been informed that the tentative date for the country's joining the EU is scheduled for the year 2050. The furious Transdniestrian government orders its only spy in Britain, Mr. G. Fawkesiuk, to blow up the House of Parliament and the Foreign Office. Having learnt about this plan, the Metropolitan police decide to cordon the area consisting of all the points x such that the distance from x to the Parliament added to the distance from x to the FO is at most 4 miles. Help the police by proving that the cordoned area is an ellipse. Find the equation of this ellipse, if the distance between the Parliament and the FO is 2 miles.