M1GLA Geometry and Linear Algebra Exercise Sheet 7

1. (a) Let s_1 be the reflection of the plane in the x_1 -axis, and s_2 the reflection in the x_2 -axis. Write down the 2×2 matrices representing s_1, s_2 and s_1s_2 , and show that s_1s_2 is a rotation. (As in lectures, s_1s_2 means the transformation given by first doing s_2 , then doing s_1 .)

(b) Let L be the straight line $x_2 = x_1 \tan \theta$, and let s be the reflection in the line L. Show that the matrix representing s is

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}.$$

(c) Let r_{θ} be the rotation through θ about the origin, and let s be the reflection in the x_1 -axis. Show that sr_{θ} is a reflection.

(d) Let L and M be straight lines through the origin, and let s and t be the reflections in L and M. Using (b), prove that st is a rotation.

2. For each of the following matrices, either find the inverse, or show that no inverse exists: (1 - 2 - 1) = (2 - 1 - 4)

$$\begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix}, \begin{pmatrix} 1 & 2 & -1 \\ 2 & -3 & 2 \\ -1 & 12 & -7 \end{pmatrix}, \begin{pmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{pmatrix}.$$

3. If A and B are invertible matrices, prove that AB is invertible and has inverse $B^{-1}A^{-1}$.

4. Let
$$A = \begin{pmatrix} 1 & 1 & 0 \\ a & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$
.

(i) For which values of a is A invertible ?

(ii) Calculate A^{-1} when it exists.

5. For 2×2 matrices A and B prove that |AB| = |A||B|.