M1GLA Geometry and Linear Algebra
Exercise Sheet 8

1. (a) Calculate $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$.

(b) Solve for t the equation $\begin{vmatrix} t-1 & 3 & -3 \\ -3 & t+5 & -3 \\ -6 & 6 & t-4 \end{vmatrix} = 0.$

(c) Solve for x the equation $\begin{vmatrix} a & b-x & c-x \\ a-x & b-x & c \\ a-x & c & b-x \end{vmatrix} = 0.$

2. For each of the following matrices A, find the eigenvalues and eigenvectors of A, and then either find an invertible matrix P such that $P^{-1}AP$ is diagonal, or prove that no such matrix P exists:

(i) $A = \begin{pmatrix} -1 & -2 \\ 4 & 5 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{pmatrix}$ (iii) $A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 3 & 1 \end{pmatrix}$.

3. Let $A = \begin{pmatrix} -10 & -18 \\ 9 & 17 \end{pmatrix}$.

(a) Find an invertible 2×2 matrix P such that $P^{-1}AP$ is diagonal.

(b) Find A^n, where n is an arbitrary positive integer.

(c) Find a matrix B such that $B^3 = A$.

(d) Find a 2×2 matrix with complex entries such that $C^2 = A$.

(e) Prove that there is no 2×2 matrix C with real entries such that $C^2 = A$.

4. Find the (complex) eigenvalues of the matrices in Question 1 (i), Sheet 6. Which of these matrices have real eigenvalues?