## M1GLA Geometry and Linear Algebra Test 1

Each question is 5 marks, provided details are given.

**1.** Write p = (3,2), a = (-1,-1). A normal vector to our line is n = (-4,5),  $||n|| = \sqrt{41}$ . Then a - p = (-4,-3), so that  $n \cdot (a - p) = 1$ . Thus the perpendicular distance is  $1/\sqrt{41}$ .

**2.** Rotate through  $\pi/4$ :

$$x = \frac{1}{\sqrt{2}}(r-s), \qquad y = \frac{1}{\sqrt{2}}(r+s).$$

We obtain  $r^2 + 3s^2 = 8$ , an ellipse.

**3.** The standard form is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where  $a = \sqrt{7}$ ,  $b = \sqrt{2}$ . Let  $(\pm p, 0)$  be the coordinates of the foci. By lectures,

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{5}}{\sqrt{7}}, \quad p = ea = \sqrt{5}.$$

By a result in lectures the directrices are the vertical lines

$$x = \pm \frac{a}{e} = \pm \frac{7}{\sqrt{5}}.$$

(Any other method is OK.)

4. Reducing the augmented matrix to echelon form we obtain

$$\left(\begin{array}{rrrr} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

Solving from back to top we find  $(x_1, x_2, x_3) = (2, -2, 1)$ .

(-2 marks if an echelon form is not shown. NB: it is not unique!)