M1GLA Geometry and Linear Algebra
Test 2. Solutions

1. 4 marks

By lectures, a solution is unique if and only if the determinant of the matrix

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & a \\ 2 & a & 4 \end{pmatrix}$$

is non-zero. This determinant is $4a - 12$. Thus a can be any number except 3.

2. 4 marks

We have $AB = BA$ if and only if

$$\begin{pmatrix} 0 & a \\ 0 & c \end{pmatrix} = \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix},$$

that is, if and only if $c = 0$ and $a = d$.

3. 6 marks.

An echelon form of the 4×8-matrix

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

is

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Clearing the columns one obtains

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Thus the right hand part of this matrix is the desired inverse.

4. 6 marks.

We need to solve the quadratic equation

$$\det \begin{pmatrix} -1-t & -2 \\ 2 & 3-t \end{pmatrix} = t^2 - 2t + 1 = 0.$$

Thus $t = 1$ is the only eigenvalue. To find the eigenvectors we solve the linear system

$$\begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0.$$

Hence the eigenvectors are the vectors of the form $c(1, -1)$, for any non-zero c.