

M1GLA Geometry and Linear Algebra Test 2

You need to show your working and justify your answers. State the results from lectures that you use.

1. Find all real numbers a such that the following system of linear equations has exactly one solution

$$\begin{aligned}x_2 + 2x_3 &= 1 \\x_1 + 2x_2 + ax_3 &= 2 \\2x_1 + ax_2 + 4x_3 &= 3\end{aligned}$$

(Any method is fine. You are *not* asked to find this solution!)

2. Find all matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $AB = BA$, where

$$B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

3. Find the inverse of the matrix

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

by reducing it to the identity matrix by elementary row operations.

4. Find all eigenvectors and eigenvalues of the matrix $\begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix}$.