M2P4 Rings and Fields
Test 3, solutions.

15 March 2007

1. **8 marks**

By Gauss's lemma $f(x) = x^3 + nx - 9$ is reducible if and only if $f(m) = 0$ for some $m \in \mathbb{Z}$. Then $m|9$ so that $m = \pm 1, \pm 3, \pm 9$. By substituting $x = m$ into $f(x) = 0$ we get the following values of n:

$$8,\ -10,\ -6,\ -12,\ -80,\ -82.$$

2. **12 marks**

Note that $x = 1$ is a root of $x^3 + \omega x + \omega^2 = 0$ since $1 + \omega + \omega^2 = 0$ in F. Thus we can write

$$x^3 + \omega x + \omega^2 = (x+1)(x^2 + ax + b),$$

for some a, $b \in F$ (recall that F has characteristic 2 so that $-1 = 1$ in F). It follows that $a = 1$ and $b = \omega^2$. The polynomial $x^2 + x + \omega^2$ has no roots in F by inspection (use that $\omega^3 = 1$), and so is irreducible in $F[x]$.