M2P4 Rings and Fields Problem Sheet 7.

1. Prove that if n > 1 and n is not prime then

$$1 + x + x^2 + \dots + x^{n-1}$$

is not irreducible in $\mathbb{Q}[x]$.

2. For which values of the integer n is $x^3 + nx^2 - 4$ irreducible in $\mathbb{Q}[x]$? Justify your answer.

3. Assume that the integers a, b, c are divisible by 4. Prove that

$$x^4 + ax^3 + bx^2 + cx + 1$$

is irreducible in $\mathbb{Q}[x]$.

4. Find the minimal polynomial of $\sin(\pi/8)$ over \mathbb{Q} .

5. Find the minimal polynomial of $4^{\frac{1}{3}}$ over \mathbb{Q} .

6. Find the characteristic of the field $\mathbb{Z}[i]/I$ where $I = (2+5i)\mathbb{Z}[i]$.

7. Let $\alpha \in \mathbb{Q}(2^{\frac{1}{5}})$. What can you say about the degree of the minimal polynomial of α over \mathbb{Q} ? Why?

8. Construct a field with 169 elements.

9. Explain how to construct a regular 12-sided polygon using ruler and compass.

10. Let C be a circle, centre X, and let P be a point outside C. Explain how to construct a tangent from P to the circle, using ruler and compass.