m3hprob3.tex

M3H PROBLEMS 3. 1.11.2013

Q1 *Archimedes' sphere-cylinder theorem.*
   Show that the surface area of a spherical slice between two parallel planes is the same as that of a circumscribing cylinder with axis perpendicular to the planes.

Q2 *Conics.*
   Show that a conic (section), defined as the Greeks did as the locus of intersection of a (doubly infinite, right-circular) cone by a plane, has an equation of the second degree in cartesian coordinates.

Q3 *Apollonius, Book III.*
(a) *The 3-line problem.* Given lines L_i ($i = 1, 2, 3$), write d_i for the distance from a point P to line L_i. Show that the locus of

$$d_1^2 = c.d_2 d_3$$

is a conic.
(b) *The 4-line problem.* Similarly with four lines: show that the locus of

$$d_1 d_2 = c.d_3 d_4$$

is a conic.

Q4 *Focus-directrix property of conics: Pappus.*
   Given a point F (the *focus*) and a line L (the *directrix*), show that the locus of points P with
$$PF = e.PL$$
is a conic (e is called the *eccentricity*).

NHB