

M3H PROBLEMS 4. 8.11.2013

Q1 *Calculation of π , after Tse Chung-chi (430-501 AD).*

(i) In a circle of radius r , let $PQ = s$ be a side of a regular inscribed n -gon. Let M be the mid-point of PQ , $u := OM$, OM produced meet the circle in R , $v := MR$, $w := RQ$. So w is the side-length of a regular inscribed $2n$ -gon (draw a diagram).

By applying Pythagoras' theorem to triangles OMP and MRQ , or otherwise, show that

$$w^2 = 2rv.$$

(ii) By taking $r = 1$, show that the iteration

$$s \rightarrow u := \sqrt{1 - \left(\frac{1}{2}s\right)^2} \rightarrow w := \sqrt{2(1 - u)}$$

takes the side of such an n -gon into that of such a $2n$ -gon.

(iii) Hence obtain π to the limits of accuracy of your pocket calculator.

Q2 (*Fibonacci sequence*).

Find the n th Fibonacci number u_n . Show that

$$u_{n+1}/u_n \rightarrow \phi := \frac{1}{2}(1 + \sqrt{5}),$$

the golden section.

Q3 (*Long division: Fibonacci (1170-1250), Liber Abaci, 1202*).

(i) If $x = m/n$ is a rational in its lowest terms, show that its decimal expansion terminates or recurs in at most $n - 1$ places.

(ii) Show that x is rational iff its decimal expansion terminates or recurs.

(iii) Find the decimal expansions of $1/7, 2/7, 3/7, 4/7, 5/7, 6/7$, and comment.

NHB