M3H HISTORY OF MATHEMATICS: QUESTIONS

Q1. *Why do dust particles dance in sunbeams?*

This phenomenon was first observed in antiquity [Week 4]: *Lucretius Carus* (c. 99 – c. 55 BC); *De rerum natura* [On the nature of things]. Lucretius' poem contains the first documented reference to (what became known in the 19th C. as) *Brownian motion*. Lucretius observed dust particles dancing in sunbeams. [I remember seeing this myself, aged c. 4, and asking my mother why they did this. Please now ask yourselves this question.]

Q2. *Why are there* **two** *tides a day?*

The Earth-Moon dynamical system is driven by Newton's Inverse-Square Law of Gravity. On Earth, the solid earth is fixed, but the sea, being liquid, is not.

Newton showed (*Principia*, 1687) that a rigid body behaves like a point mass concentrated at the centre of mass (this needs integral calculus, which Newton developed in the *Principia*). Why does the Moon always show the same face to the Earth, rather than revolve on its axis as the Earth does?

Imagine a pair of young lovers on a beach, with a full Moon overhead. They see a high tide, for an obvious reason: the Moon overhead *pulls* the sea in front of them *upwards*.

Now imagine a young couple on the far side of the Earth (the Antipodes), again on a beach. The Moon is directly underneath them, so invisible through the Earth. But again they see a high tide, this time for a non-obvious reason. The Earth behaves like a point mass at its centre. This is nearer to the Moon than the beach (by the radius of the Earth), so it feels the Moon's gravity more than the sea in front of the beach. The lovers see a high tide because the sea-bed in front of them is *sucked downwards*.

This phenomenon had been a standing puzzle for long before Newton – who was the first person to understand it. This alone partly explained the enormous impact of the *Principia*. Note that it can be explained in words – no mathematics needed – once one has the concepts in place.

Q3. *Why is the sky blue?*

Lord Rayleigh (J. W. Strutt) (1842-1919), 7th President LMS (1876-8), PRS

(1905-8), Nobel Prize 1904.

The theory of sound, vol. I, II, 1877 (Dover, 1945)

Rayleigh's work on *scattering* explained why the sky is blue (the earth's atmosphere scatters sunlight – the sky is black in space). For, the scattering is done by photons in the Sun's light interacting with the electric forces within the atoms of the dust particles. Recall that (Maxwell) light is electromagnetic. The scattering is done differentially according to wavelengths. The red end of the visible spectrum is scattered more than the blue end, which thus survives to strike the eye.

Rayleigh's Nobel Prize was for the discovery of the inert gas argon.

Q4. *Why do we sweat?*

We sweat because we are warm-blooded. Our bodies are thus very sensitive to temperature. We sweat to cool down – just as we shiver to warm up, to get our muscles moving.

When we sweat, the sweat drops have surface tension, as with any water surface. The water molecules are in thermal oscillation, or bombardment. They have a temperature distribution – the *Maxwell-Boltzmann distribution* (Week 9) of Statistical Mechanics. It is only the more energetic (fast-moving, hotter) molecules that have enough momentum to break out of the drop against the force of the surface tension. Thus evaporation of sweat cools one by draining the sweat differentially of its 'hotter' molecules, so we lose heat and cool down.

Note that this does not work if the air is *humid*, as then it is harder for evaporation to take place. We still lose water through sweat (so risk becoming dehydrated), but do not get the benefit of the cooling (so our core temperature will rise, if we are taking strenuous exercise). This is why humid conditions are feared by endurance athletes such as marathon runners: this can lead to heat-stroke.

Q5. *Why does water boil?*

As in Q4: at a high enough temperature, most of the water molecules are energetic enough to break through the water surface.

Q6. *Why does water freeze?*

When water freezes – to ice, or snow – it does so into crystals, which are ordered. It is easier for the slower-moving (so colder) water molecules to attach themselves to these crystals. So, as in Q5, these are differentially removed from the water – which is thus warmed, by their departure (as in Q4, but with the direction of temperature reversed).

It is easier for water to freeze when there is some existing ice – or even dust – to 'freeze onto', a phenomenon known as nucleation. If rain falls through dust-free air, it may not freeze even though the temperature is well below freezing-point. When this happens, it falls as rain rather than hail or snow – but instantly freezes on impact. The trees then become overloaded with an increasingly thick coating of ice, and fall under the weight – an *ice storm*, common in the USA and Canada.

Q7. *What causes a rainbow?*

The rainbow is formed when the sun is behind the observer, and rain is falling ahead of the observer. One needs two physical principles:
(a) The law of reflection: when light is reflected at a mirror, the angle of incidence = angle of reflection.
(b) Snel's law of refraction: "mu sin theta = constant", where mu is the refractive index (higher for water than for air, as light travels more slowly in water than in air), and theta is the angle of incidence): the Dutch scientist Willebrord Snel (1581-1625), 1618 and 1621. The rainbow has a definite 'size'. This is the angle between the line L_1 from the Sun through the observer and any line L_2 from the observer to the arc of the rainbow (any such line gives the same angle, as this arc is circular).

The rainbow is produced when light from the Sun is
(i) *refracted* when it enters falling raindrops in front of the observer (and is bent *towards* the normal);
(ii) *reflected* at the back of the raindrop (which acts as a mirror);
(iii) *refracted* again when it exits the raindrops.

A visible effect – the rainbow – is obtained when the angle of deviation has an *extremum* (minimum). For here, many rays will emerge parallel.

Light of different colours (wavelengths) have different refractive indices, so are separated by the two refractions, and one sees the colours of the rainbow. The red end of the visible spectrum subtends an angle of 42.1^o, the violet end an angle of 42.1^o. This is the *primary rainbow.*

The first qualitatively correct explanation is due to *M. A. de Dominis* (1564-1624) in 1611 (there is also work by Kepler in 1611).

This was taken further by Snel in 1618 and 1621.
René Descartes (1596-1650); *Discours de la Méthode ...*, 1637; (first appendix, *La Géométrie*, cartesian geometry;) second appendix, *La Dioptrique.* This

gave a treatment of the rainbow, including an estimate of the angle, 42^o. Descartes did not have calculus, so could not give an analytic solution.
(Sir) Isaac Newton (1642-1727). Newton lectured in Cambridge (1669-71) on the rainbow, and wrote a paper for the Royal Society in 1672 on the composite nature of white light. His *Principia* of 1687 contains all this.

Source: C. B. BOYER, *The rainbow: From myth to mathematics*, Princeton UP, 1987 (1st ed. 1959).

Q8. *What causes a double rainbow, and how does it differ from a single one?*

With a *secondary rainbow*, there are *two* reflections rather than one. The extra reflection both reverses the order of the colours, and has is fainter (light loses intensity when reflected – one can see this on shining a torch beam at a mirror). The secondary bow is larger (adapting the mathematics needed to calculate the angle of the primary bow – see Boyer above, or a textbook on Optics).

Q9. *What is a sonic boom, and what causes it?*

A sonic boom is a shock wave, caused by a moving body travelling faster than the speed of sound. So the body 'gets ahead of its own sound'. Typical examples are a rifle bullet (hence the characteristic crack/whistle), and a supersonic jet (only military nowadays, since Concorde went out of service).

As the body accelerates through the 'sound barrier', the relevant PDE changes from elliptic (as with Laplace's equation) through parabolic (as with the heat equation) to hyperbolic (as with the wave equation here).

Q10. *What causes partial reflection of light at a mirror?*

The light ray is a stream of photons – particles, radiation. The mirror is matter. The interaction of matter and radiation is the subject of Quantum electrodynamics (QED). See e.g.
R. P. FEYNMAN, *QED: The strange theory of light and matter*, Princeton UP, 1985.

Newton studied partial reflection of light at a mirror in his *Opticks* of 1704. We now know it is a *quantum* phenomenon. Newton had no hope of solving it long before the quantum age began with Planck in 1900. What impressed Feynman is that Newton recognised that there is a deep mystery here.

NHB