m3hw5.tex Week 5. 5.11.2013

FROM THE ARABS TO GALILEO

Edward GIBBON, *Decline and fall of the Roman Empire*, 1776
Boyer Ch. 14; Dreyer Ch. X, Mediaeval cosmology

Al-Kwarizmi's Algebra (concluded). Its strengths include logical exposition and proof (shared by the Greeks but *not* the Mesopotamian and Hindu influences), and facility in passing between geometric (Greek) and numeric (Hindu) algebra. Its weaknesses include that only positive rots of equations were permitted, and that everything, even numerals, is written out in words.
Astronomy

The Caliph al-Mamun founded an observatory in Baghdad in 829, 'where continuous observations were made and tables of planetary motions constructed, while an important attempt was made to determine the size of the Earth' (Dreyer, 246). One of the leading astronomers here was *al-Fargani* (= Alfragamus), 'whose *Elements of Astronomy* were translated into Latin in the 12th C., and contributed greatly to the revival of science in Europe.'

Hindu astronomy was derived from the Greek through the conquest of Alexander the Great (B 12.12). Hindu astronomical texts, the *Siddhanthas*, reached Baghdad during the reign of Caliph al-Mansur 'in 773) (Dreyer, 244), or 'by 766' (B, 254). These were translated into Arabic as the *Sindhind*.
Note. One of the main spurs behind Arab interest in astronomy derives from the Muslim religious year.
Omar Khayyam (c. 1050-1123) (B 13.15).

Kayyam wrote a text *Algebra*. Like al-Khwarizmi, he restricted himself to *positive* roots, but went beyond degree 2. He gave algebraic and geometric solutions for quadratics, and geometric solutions (involving intersecting conics) for cubics. He was also a noted poet. His poem *The Rubaiyat of Omar Khayyam* was translated into English poetry by Fitzgerald in 1872/79/89.
Postscript to 'pre-Europe'.

Mathematics is an intensely *cultural* activity – because unless one builds on the work of others, one will come much less far in one lifetime than humanity has come already! From now on, we enter the framework of *European* culture (apart from N. American in the 20th C.). We do this for brevity, not cultural bias – ancient and non-European cultures produced far more than we have time to touch on. But accusations of cultural bias do exist, and 'ethnomathematics' is sometimes advocated as an antidote. See e.g. G. C. JOSEPH, *The crest of the peacock: Non-European roots of mathematics*, I. B. Tauris, 1991. Rev.: Amer. Math. Monthly 99.7 (1992), 692-4; Marcia ASCHER, *Ethnomathematics: A multicultural view of mathematical ideas*, Brooks Cole, 1991.

Background: pre-Europe to Europe

Historically, one speaks of ancient, or classical, history, the Middle Ages or mediaeval period, and modern history. Customarily, the Middle Ages are taken from the fall of Rome in 476[1] to the fall of Constantinople in 1553. Mathematically, we take the end of the classical period as 529, when Justinian closed the Academy at Athens. The surviving scholars despersed East to Syria and Persia, where they eventually encountered Arab scholarship.

The early mediaeval period is often called the Dark Ages by historians, and this is apt in mathematics too. In West Europe, the dominant institutions (which often quarreled) were the Roman Catholic Church, led by the Pope as Bishop of Rome, and the Holy Roman Empire (which lasted from Charlemagne in 800 to Napolean in 1804). Throughout the Middle Ages, the great majority of educated people were clerics, monks et. (ordinary people, and many political leaders, were illiterate), and the language of learning was Latin. The subjects of learning were Christian theology (something of a minefield – one had to beware of being accused of heresy), works of the classical Latin authors, etc. (a revival of interest in classical Greek authors came later), rather than science. The Church remained suspicious of science as late as Galileo's time, c. 1600 (and indeed, later).

In the East, the energies of the Byzantine Empire during its last eight centuries were absorbed by its struggles with Islam[2], and with the West. Greek Orthodox Christianity was considered partly heretical by the Roman Catholic Church[3], and Constantinople was sacked by Crusaders during the Second Crusade. It never recovered, though it survived till 1453.[4] Byzantium

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[1]Recall that Rome was sacked by Alaric the Visigoth in 410; 476 marked the final dissolution of the Roman Empire in the West, when Emperor Romulus Augustus was deposed by Odoacer, a Germanic chieftain.

[2]Arabs initially, later the Ottoman Turks

[3]The differences centred on the divinity of Christ, and came to a head over the presence or absence of the word *filioque* – and of the Son – in liturgy.

[4]330-1453 – 1,123 years – is not bad for a human institution. When Hitler began WWII by invading Poland, he announced 'The fighting that begins today decides the future of the German-speaking people for the next thousand years'. His 'thousand-year Reich' did not happen – but Constantinople did even better.

contributed nothing original to mathematics (B 14.2); its only importance lay in its preservation of Greek texts, and commentaries on these, such as those of Proclus (B 11.13).

The 'Golden Age of Islamic Mathematics' resulted in the spread of Arab mathematic – algebra, trigonometry, astronomy etc. – from Baghdad and Persia in the East, along N. Africa, to Spain in the West. A particularly important centre of learning was Cordoba, which produced several noted astronomers (Dreyer, 262) and the Jewish scholar Maimonides (1135-1204 – see W for details). Contact between the Christian and Muslim worlds were mainly through Spain, Venice, Constantinople and Sicily (a Muslim province at that time).

Translation

Toledo had been for centuries part of Muslim (Moorish) Spain (al-Andalus, hence Andalucia), but was reconquered by Christendom. The Archbishop was enlightened enough to approeciate the Islamic cultural heritage, and encouraged the translation of Arabic works into Latin. Toledo, which had excellent libraries and scholars of all religions and languages, became a centre for translation. In particular, the extensive mingling over long periods of time between Muslim, Jewish and Christian scholars did much to prepare the ground for the great flowering that marked the take-off point for European culture and learning – including mathematics and science – the Renaissance (below).

Adelard of Bath (c. 1075-1160).

Adelard (Athelhard) produced the first translation of Euclid's *Elements* from Arabic to Latin in 1142 (Heath I, 362-3). A passage of Old English verse quoted by Heath puts the introduction of Euclid into England as far back as King Athelstan's reign (924-939).[5]

Fibonacci, Leonardo of Pisa (c.1180-1250) (B 14.6-10)

Leonardo of Pisa, son of Bonaccio (hence 'Fibonacci') wrote the *Liber Abaci* (Book of the Abacus) in 1202. This was the most influential European mathematical work before the Renaissance, and was the first such book to stress the value of the (Hindu-)Arabic numerals (Fibonacci had studied in the Muslim world and travelled widely in it).

The *Fibonacci sequence* $u = (u_n)_{n=0}^{\infty}$, 1,1,2,3,5,8,13,21,... (B 14.8) is gen-

[5]Athelstan (c.894-939) was the first King of England, bringing Northumbria, Mercia and Wessex together politically for the first time with his conquest of Viking York in 927. Offa, Alfred the Great and Athelstan are regarded as the three greatest Anglo-Saxon kings.

erated by the recurrence relation

$$u_n = u_{n-1} + u_{n-2} \qquad (u_0 = u_1 = 1).$$

It occurs naturally in various problems of growth.

The Rise of Universities

The academies (Pythagorean, Athenian, Alexandrian) of the ancient world played the role of universities in their day, as did their Arab counterparts in Baghdad, Cordoba etc. By the 12th C., the modern concept of a university as an autonomous academic institution awarding degrees, and as a centre of learning, teaching and research, began to emerge. This was a gradual process. The earliest continental universities are Bologna (founded 1088; Royal Charter 1130), Salamanca (founded 1134, from a Cathedral School, 1130; Royal Charter 1218) and Paris (mid-12th C.). In Britain, the universities of Oxford and Cambridge simply describe themselves as 'founded in the 12th C.' and 'founded in the 13th C.' respectively. Scotland has four ancient universities: St. Andrews, 1410; Glasgow, 1451; Aberdeen, 1495; Edinburgh, 1583. Ireland has Trinity College, Dublin, 1592. London has UCL, 1826, and KCL, 1828-9; the University of Durham was founded in 1832 and the University of London (incorporating UCL and KCL) in 1836. Imperial College London was founded in 1907 (and left U. London in 2007).

Latin

We have commented repeatedly on the adverse effect on mathematics of the Roman influence. Here at last we see the positive side of the Roman bequest to mathematics, and to science and learning generally: *Latin.* The Latin language was the common medium of communication among scholars from the Dark Ages on, through the rise of the universities, for several centuries,[6] till the gradual replacement by (mainly) French, German and English. Thus Newton and Euler wrote in Latin, while the mathematicians of the French Revolution used French. Gauss began with Latin and ended with German. Scientific Latin survives today in biology (names of species and genera, e.g. in the classification by Carl Linnaeus (1707-1778), medicine (names of diseases), etc.[7]

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[6]Queen Elizabeth I of England once conducted a spirited political argument with the Polish Ambassador – in fluent Latin.

[7]Latin was compulsory for entrance to Oxford and Cambridge in my day. I studied classical Latin for O Level (GCSE came later), and remember studying Newtonian Latin in the Scholarship Sixth (in those days, Oxbridge entrance was held at Christmas after A Level, so applicants had to at least begin a third year in the Sixth, now almost unknown).

THE RENAISSANCE TO GALILEO

Background

The Renaissance, or re-birth, marks the end of the Dark Ages and the re-emergence of European culture dormant since the classical period. The history of mathematics, and science, will be primarily concerned with Europe (and later, its offshoots in America) from now on.

The Renaissance (the term dates back only to 1855, in Michelet's *Histoire de France*) is a broad term covering the 14th-17th centuries, but is regarded as having begun in Florence in the 14th C. One factor here was the role of the Medici family, who were originally bankers before going into politics. Banking and finance (of which more below) flourished on contact between civilisations, which tended to have a cross-fertilising effect; also, the Medici's money enabled them to become great patrons of the arts. Later, the influx of Greek scholars after the Fall of Constantinople in 1453, bringing with them many texts and much learning, was also an important factor.

Perspective

As we have seen, perspective was known (at least in part) in the ancient world, but was then lost.

Filippo Brunelleschi (1377-1446) discovered the main principle of perspective – the use of vanishing points – and convinced his fellow-artists of this in a famous experiament of 1420 involving the chapel outside Florence Cathedral.

Leon Battista Alberti (1404-72), *Della pictura* (1435, printed 1511) gave the first written account of perspective.

Piero della Francesca (1410-92), *De prospectivo pingendi* (c. 1478). In his book, and in his painting, Piero della Francesca did much to popularise perspective, which spread throughout the Western art world.

Leonardo da Vinci (1452-1519); *Trattato della pittura*. Leonardo is usually regarded as the personification of Renaissance genius. He was a prolific inventor, an artist who wrote on perspective, and a mathematician.

Albrecht Dürer (1471-1528) of Nuremburg; *Investigations of the measurement with circles and straight lines of plane and solid figures* (1525-1538, German and Latin). Like Leonardo, Dürer was both a mathematician and an artist. He adopted perspective after visiting Italy.

Source: E. C. ZEEMAN, The discovery of perspective in the Renaissance (LMS Popular Lecture, 1983).

Printing and books

An important turning-point was the invention of the printing-press. Movable type was introduced by Johannes Gutenberg (1395-1468) of Mainz in Germany (where the University is named after him), in his Bible of 1455. The first printed book in English (on the Trojan War) was produced by William Caxton in Bruges in 1476 (he then moved to London and established the first printing press in Britain). Printing and books enormously increased the scope for the rapid dissemination of knowledge.

Universities

Learning was now dominated by the universities, focussing on mediaeval Latin for theology etc. and Latin translations from the Arabic in science. Printing and books led to more emphasis on Greek culture, both in literature and in science. The Greek classics in both could now be read in the original, translated directly (from the Arabic), printed, and distributed widely.

Regiomontanus (1436-76) (B 15.2-5)

Born Johann Müller of Königsberg (the german city in East Prussia founded by the Teutonic Knights, now Kaliningrad in Russia), he was known as Regiomontanus (= king's mountain in Latin, = Königsberg in German),

Almagest. Regiomontanus completed a new Latin translation of Ptolemy's *Almagest* (begun by Peuerbach), which – with its commentary – was mathematically superior to previous versions.

De triangulis omnimodis (1464). Probably influenced by the work of the Arab mathematician Nasir Eddin, this was the first major European work on trigonometry, and assisted in the evolution of trigonometry as a subject in its own right, independent of astronomy.

Book I: Solution of triangles; Book II: Sine rule; Book IV: Sine rule for spherical triangles.

Nicholas Chuquet (fl. c. 1500)

Triparty en la science des nombres, 1484 (B 15.6): the most important European mathematical text since the *Liber Abaci*.

Part I: Hindu-Arabic numerals; addition, subtraction; multiplication; division; Part II: Surds; Part III: Algebra; laws of exponents; solution of equations.

Luca Pacioli (1445-1514)

Summa de arithmetica, geometrica, proportioni et proportionalita, 1494 (B 15.7). This was an elementary text (more influential than the *Triparty*) on arithmetic, algebra and geometry. It is notable for *double-entry book-keeping*, and use of the *decimal point*.

Geronimo Cardano (1501-76); *Ars Magna*, 1545 (B 15.10-13).

Cardano's *solution of the cubic* (published here in 1545) marks 'the be-

ginning of the modern period in mathematics'. Scipione del Ferro (c. 1465-1526), Professor of Mathematics at Bologna, solved the cubic but did not publish his results. Niccolo Tartaglia (c. 1500-1557) (b. Niccolo Fontana; Tartaglia = stammerer), knowing of del Ferro's solution, found one himself, by 1541. Predictably, this led to a priority dispute with Cardano.
Complex numbers. Cardano, in Ch. 37 of *Ars Magna*, solves

$$x(10 - x) = 40,$$

obtaining roots $5 \pm \sqrt{-15}$, and notes that

$$(5 + \sqrt{-15})(5 - \sqrt{-15}) = 25 - (-15) = 40.$$

Thus (Kline, 253) 'Without having fully overcome their difficulties with irrational and negative numbers, the Europeans added to their problems by blundering into what we now call complex numbers.' Though complex numbers were not properly assimilated into mathematics till much later, they enter the stage with *Ars Magna*.

As Boyer notes (p.322), 'whenever the three roots of a cubic are real and non-zero, the Cardan-Tartaglia formula leads inevitably to square roots of negative numbers'. For a detailed account of this 'irreducible case' of the cubic, and of the quartic, see e.g.
W. L. FERRAR, *Higher algebra*, OUP, 1943; Ch. XXI.
Quintics and polynomials of higher degree

The solution of cubics and quartics prompted an attack on the quintic and polynomials of degree at least 5. What was sought was a 'solution by radicals': a *formula* for expressing (or an algorithm for finding) the roots in terms of the coefficients. As we shall see later:
(i) Although a polynomial of degree n does indeed have n roots (possibly complex, counted according to multiplicity) (Fundamental Theorem of Algebra: Complex Analysis, 19th C.), (ii) If $n \geq 5$, no solution by radicals exists (Abel, Galois, Algebra – Field Theory; 19th C.).
The quintic here joins the classical Greek problems (circle-squaring, cube-duplicating, angle-trisecting and the proof of the parallel postulate) among the Holy Grails of mathematics. None of these problems is soluble, but, the search for a solution led to other things.

Cardano (c. 1526) wrote a book *De Ludo Aleae* (On Dice Games), published posthumously in 1663. This was the first book written (though not the first book published) on Probability Theory.

Nicholas Copernicus (1473-1543) of Thorn (Niklas Koppernigk of Torun, Poland) (B 15.15); *De revolutionibus orbium coelestium*, 1543.

This work revolutionised astronomy by expounding the *heliocentric theory* – that the earth and other planets revolve around the sun. With Copernicus, the modern period of astronomy begins. See Dreyer Ch. XIII for a detailed account of this epoch-making achievement (and Dreyer Ch. VI and Week 2 for a discussion of Aristarchus and the heliocentric theory).

Astronomical measurement. Copernicus measured the distances from the Earth and other planets to the Sun. His figures are close to those of Ptolemy.

De lateribus et angulis triangulorum, 1542: a book on trigonometry, probably influenced by Regiomontanus.

Cartography

Recall that in 1486 the Portuguese reached India by sailing East round the Cape of Good Hope (the Portuguese lead was thanks to Henry the Navigator, d. 1460), and that in 1492 Columbus discovered America by sailing West. These developments increased demand for new maps, and better techniques of map-making.

Gerard Mercator (1512-1594), Flemish geographer

Mercator's projection (1569): project the Earth onto a circumscribing cylinder with axis due North; cut and unwrap. This works well *except* near the Poles, where there is gross distortion. Compare with *stereographic projection*, due to Ptolemy and still used in Complex Analysis. Of course, one cannot hope to represent a sphere on a plane without introducing a singularity in the representation somewhere, because of their topologically different natures.

Aside: business and finance

The growth of commerce was an important factor in the transition from mediaeval to modern times. It encouraged the replacement of a feudal society, based on land (where a tenant held land 'in feu' or on trust (foi = faith) from the king or lord in exchange for military service in time of war) to modern society based on commerce, trade and money. Double-entry bookkeeping is needed to keep adequate accounts. Every transaction occurs twice, once as a credit and once as a debit; the sums of credits and sums of debits have to balance. The accounts are displayed in a *balance sheet* (spreadsheets developed from these); the profit (or loss) appears in the bottom right-hand corner (hence the expression "bottom line").

We note the establishment of the Royal Exchange in London in 1570 by Sir Thomas Gresham, under Elizabeth I.

Francois Viète (1540-1603) (Franciscus Vieta) (B Ch. 16)

Viète was a lawyer, and councillor to King Henry IV of France (Henri de Navarre). He was also a successful cryptographer.

Decimals. Viète advocated decimals rather than sexagesimals, and was partly responsible for the evolution of the notation for the decimal point (Cajori, p.316).

Parameters. Algebraic notation of that time was still very clumsy by modern standards. Viète used vowels to represent unknowns, consonants for coefficients (or parameters) – an important notational advance for its time. Compare the modern practice: unknowns x, y, z near the end of the alphabet, coefficients a, b, c near the beginning.

Algebra (B 16.3): *Isagoge* (Introduction), 1591.

Cubics (B 16.4). Viète introduced the substitution normally used today for solution of cubics, in preference to the Cardano-Tartaglia method. See e.g. G. BIRKHOFF & S. MACLANE, *A survey of modern algebra*, Macmillan, 1941/53, V.4,5,6.

Prosthaphaeresis (Greek: 'addition and subtraction'). The formulae

$$2\sin A\cos B = \sin(A+B)+\sin(A-B), \quad 2\cos A\cos B = \cos(A+B)+\cos(A-B),$$

$$2\cos A\sin B = \sin(A+B)-\sin(A-B), \quad 2\sin A\sin B = \cos(A+B)-\cos(A-B)$$

are variants on Ptolemy's formulae (sometimes called *Werner's rules*, after Johannes Werner (1468-1522), B 15.19) enable *multiplication* (the operation on LHS) to be replaced by addition, subtraction and use of trigonometric tables. This procedure seems tortuous to the modern eye, but was found to be of great practical value in avoiding the drudgery of long multiplication, in this period after the construction of excellent trigonometric tables but before the introduction of logarithms (below).

Multiple-angle formulae. Viète also showed that $\cos n\theta$ is a polynomial of degree n in $\cos\theta$ (and $\sin n\theta$ is $\sin\theta$ times one of degree $n-1$). These polynomials are those now known as the *Tchebyshev polynomials* T_n, U_n of the first and second kinds.

Trigonometric solution of equations. Viète's trigonometric solution to the cubic extends to higher degree; he used it for degree 45 in 1593.

Infinite product for π. Viète produced the first exact expression for π in closed form (B 16.16), and calculated π to 10 sig. fig.

Albert Girard (1590-1633) of Flanders (B 16.4)
Invention nouvelle en l'algèbre (1629): Negative and imaginary roots of polynomials; irreducible case of the cubic; sums of roots, of squares of roots, etc. Girard also conjectured the Fundamental Theorem of Algebra.
Girard's formula, or the *formula of spherical excess*

For a spherical triangle with angles A, B, C, $A + B + C > \pi$, and $A + B + C - \pi$ is called the *spherical excess*. If the sphere has radius r, then the area Δ of the triangle is given by

$$\Delta = r^2(A + B + C - \pi).$$

Note: if the area is small for fixed r, then so is the excess. So for small spherical triangles on the earth's surface, the sum of the angles is approximately π. This just says that we may neglect the Earth's curvature for triangles small in relation to the Earth, and this is of course used in map making (recall the trig points on OS maps!).
John Napier (1550-1617) and *logarithms* (B 16.9,10)
Mirifici logarithmorum canonis descriptio (1614),
Mirifici logarithmorum canonis constructio (1619, posth.)

Napier, a Scottish baron, was led to his discovery of logarithms (logos = ratio + arithmos = number) through being told (by Craig) of the use of prosthaphaeresis (by Tycho Brahe – see below – in Denmark). His logarithms were an early form of modern logarithms to base 10 (he did not have the number e, which needs calculus). [8]
Henry Briggs (1561-1631) (B 16.11). Briggs was the first holder of each of the two earliest chairs of mathematics to be founded in Britain: that at Gresham College, London (1596)[9] and the Savilian Chair of Geometry at Oxford (1619). Briggs also introduced the method of *long division* still in use today.
Simon Stevin (1548-1620) of Bruges, Flemish military engineer (B 16.13,14).
Die Thiende, *La disme* ('the tenth'), 1585.

Stevin's elementary book did more than any other to popularise the use of decimal fractions (decimals), and to spread awareness of their computational value and superiority.

[8] In my time at school, logs to base 10 were used up to O Level (the GCSE of its day). Use of a slide rule was a Sixth former's privilege. The pocket calculator arrived in the early 70s.

[9] Gresham College still exists, on the S side of High Holborn near Chancery Lane tube. Gresham lectures are open to the public, and recommended.

Thomas Hariot (1560-1621); *Artis Analyticae Praxis* (1631, posth.) (B 16.5).

Kline writes (p.280): 'Hariot's *Praxis* extended, systematised and brought out some of the implications of Vieta's work. The book is much like a modern text on algebra; it is more analytical than any algebra preceding it, and presents a great advance in symbolism. It was widely used.' Hariot also introduced the inequality signs $<$, $>$ (the equals sign $=$ had been introduced by Robert Recorde in his *Ground of Artes* in 1541). The book would have been even better, had his executors not omitted the parts they did not understand.

Hariot was mathematics tutor, and later navigator and astronomer, to Sir Walter Raleigh. He accompanied Raleigh to America (1585-6), visiting Roanoke Island, N. Carolina. He was the expedition interpreter, having previously learned Algonquin from two Algonquins in England. He was the first to make a drawing of the Moon through a telescope (26.7.1609, four months before Galileo), and the first to find the formula of spherical excess (though Girard published it first).

William Oughtred (1574-1660), a London clergyman; *Clavis Mathematicae* (Key to Mathematics), 1631 (B 16.5).

Oughtred introduced the multiplication sign $\times$ (St. Andrew's cross). His *Clavis* was one of the books from which Newton learned mathematics.

Galileo Galilei (1564-1642) (B, 16.15,20,21)

'Galileo was the first truly modern scientist – the first whose outlook and methods would not be out of place even today. He made important discoveries in astronomy and mechanics, but his greatest achievement was the creation of the experimental-mathematical method that has lain at the basis of all progress in physical science since his time'.

Professor of Mathematics at Pisa, 1589. He demonstrated that heavy and light weights fall at the same speed by dropping them from the Leaning Tower of Pisa. His experimental approach to science brought him into conflict with the Church (below). He was driven out of Pisa in 1592, going to Padua as Lecturer in Mathematics. He eventually returned to Pisa, becoming Philosopher and Mathematician to the Grand Duke of Florence.

Astronomy.

Galileo invented (as well as an air thermometer) a telescope. With this, he began observations in 1609, observing

(i) the Mountains of the Moon;

(ii) the four Moons of Jupiter;
(iii) the phases of Venus (incompatible with the geocentric system, since this shows that Venus orbits round the Sun).
These discoveries provided crucial observational support for the Copernican theory.
The Two Chief Systems (1632). Written (like Plato's dialogues) in the form of a dialogue between three characters, this book supported the Copernican keliocentric theory, and brought Galileo into conflict with the Inquisition. Under threat of torture, in 1633 he was forced to disavow it. The (Roman Catholic) Church admitted that it was wrong to condemn Galileo in 1992, 359 years later.
The Two New Sciences (1638): on mechanics. He showed that
(i) a body falling under gravity does so with constant acceleration;
(ii) the trajectory of a projectile is a parabola.
Although the ancient Greeks had an excellent knowledge of conics, they were capable of asserting that projectiles describe arcs of circles! This illustrates both Greek limitations and the power of Galileo' new scientific method.
Infinite sets. Galileo noticed the characteristic property of an infinite set: it can be put into one-one correspondence (bijection) with a proper subset of itself (e.g., $\mathbb{Z} \leftrightarrow 2\mathbb{Z}$ under $n \leftrightarrow 2n$). This was taken up in the 19th C. by Dedekind.
Epitaph. Galileo is reported to have withdrawn his forced recantation on his deathbed with the words 'E pur si muove' ('And still it moves'), which, apocryphal or not, may serve as an epitaph to one of the giants of science.
Sir Francis Bacon, 1st Viscount St. Alban (1561 - 1626).

W: "... he remained extremely influential through his works, especially as philosophical advocate and practitioner of the scientific method during the scientific revolution. Bacon has been called the creator of empiricism. His works established and popularized inductive methodologies for scientific inquiry, often called the Baconian method, or simply the scientific method. His demand for a planned procedure of investigating all things natural marked a new turn in the rhetorical and theoretical framework for science, much of which still surrounds conceptions of proper methodology today."[10]

[10]Bacon lived at Gorhambury, just N. of St. Albans, Herts, near the Roman Theatre. The house is open (National Trust) – recommended.