

**M3H PROBLEMS 4. 5.2.2016**

Q1 *Calculation of  $\pi$ , after Tse Chung-chi (430-501 AD).*

(i) In a circle of radius  $r$ , let  $PQ = s$  be a side of a regular inscribed  $n$ -gon. Let  $M$  be the mid-point of  $PQ$ ,  $u := OM$ ,  $OM$  produced meet the circle in  $R$ ,  $v := MR$ ,  $w := RQ$ . So  $w$  is the side-length of a regular inscribed  $2n$ -gon (draw a diagram).

By applying Pythagoras' theorem to triangles  $OMP$  and  $MRQ$ , or otherwise, show that

$$w^2 = 2rv.$$

(ii) By taking  $r = 1$ , show that the iteration

$$s \rightarrow u := \sqrt{1 - \left(\frac{1}{2}s\right)^2} \rightarrow w := \sqrt{2(1 - u)}$$

takes the side of such an  $n$ -gon into that of such a  $2n$ -gon.

(iii) Hence obtain  $\pi$  to the limits of accuracy of your pocket calculator.

Q2 (*Fibonacci sequence*).

Find the  $n$ th Fibonacci number  $u_n$ . Show that

$$u_{n+1}/u_n \rightarrow \phi := \frac{1}{2}(1 + \sqrt{5}),$$

the golden section.

Q3 (*Long division: Fibonacci (1170-1250), Liber Abaci, 1202*).

(i) If  $x = m/n$  is a rational in its lowest terms, show that its decimal expansion terminates or recurs in at most  $n - 1$  places.

(ii) Show that  $x$  is rational iff its decimal expansion terminates or recurs.

(iii) Find the decimal expansions of  $1/7, 2/7, 3/7, 4/7, 5/7, 6/7$ , and comment.

NHB