m3hprob5.tex

M3H PROBLEMS 5. 12.2.2016

Q1 (*Polar equation of a conic*).

Show that the equation of a conic with focus the origin, directrix the line $L : y = \ell$ and eccentricity e is

$$\frac{1}{r} = \frac{1}{\ell}(1 + e\cos\theta).$$

Q2 (*Inverse Square Law of Gravity and conical orbits*).
(i) Find the components of velocity and acceleration along and perpendicular to the radius vector in polar coordinates.
(ii) Show that the force being *central* (along the radius vector) is equivalent to *Kepler's Second Law*: the radius vector sweeps out equal areas in equal times.
(iii) Show that *Newton's Inverse Square Law of Gravity* is equivalent to the differential equation

$$\frac{d^2u}{d\theta^2} + u = b. \qquad\qquad (DE)$$

Solve (DE), and identify its solution as a conic.
(iv) Which authors and periods are involved in your proof?

<div align="right">NHB</div>

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