m3hw2.tex **Week 2 20-22.1.2016**

THE GREEKS: THALES TO EUCLID

Sources:
B, Ch. 4-11;
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D. H. FOWLER, *The mathematics of Plato's Academy: A new reconstruction*, OUP, 1987 (pb 1990).
B. L. van der WAERDEN, *Geometry and algebra in ancient civilizations*, Springer, 1983;
Otto E. NEUGEBAUER, *The exact sciences in antiquity*, Princeton UP, 1952; 2nd ed. Brown UP, 1957/Dover, 1969.
J. L. E. DREYER, *A history of astronomy from Thales to Kepler*, CUP, 1906 (revised with a forewrod by W. H. Stahl, Dover, 1953).
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H. S. M. COXETER, *Regular polytopes*, Macmillan, 1963; Dover, 1973.

Dramatis Personae
Thales (c. 624 – 548 BC) of Miletus (Asia Minor)
Pythagoras (c. 580-500 BC) of Samos (Dodecanese island)
Anaxagoras (d. 428 BC) of Klazomenae (Ionian coast, Asia Minor)
Zeno (fl. c. 450 BC) of Elea (now Italy)
Hippasus (fl. c. 400 BC) of Metapontum (Italy)
Democritus (c. 460- ? BC) of Abdera (Thrace)
Socrates (469 – 339 BC)
Plato (428 – 348 BC)
Eudoxus (of Cnidus, 408 – 355 BC)
Menaechmus (fl. c. 350 BC)
Aristotle (384 – 322 BC)
Euclid (of Alexandria, fl c. 300 BC)

We turn now from the pre-Greek world of mathematics to the classical period (c. 800 BC – c. 800 AD; B, Ch. 4-11), or from the potamic (river-based) to the thalassic age (thalassa = sea, Greek).

The Greeks, or Hellenes, established themselves not only in Greece, but in many parts of Asia Minor (Anatolia – modern Turkey), Egypt and Italy. They showed unsurpassed intellectual energy: beginning from nothing, they absorbed the wisdom of the ancient world, which they came into contact with through their trading activities. Building on this and re-working it, they then far outstripped the achievements of their predecessors.

Computation

Everyday reckoning of this period – such as in business, at which the Greeks excelled (B, 66) – employed an abacus of some kind. Such practical skills were regarded as mundane by the early mathematicians, who used the term 'logistic' for it, reserving 'arithmetic' for what we call the Theory of Numbers, or Number Theory ['arithmetic': the science of number, from the Greek for number; 'logistic': reckoning or calculating, from the Greek 'to reckon'. 'Logistics' originally meant the moving or quartering of troops, from the French 'loger', to quarter].

Thales of Miletus (c.624 – 548 BC; Asia Minor).

'To Thales, the primary question was not *what do we know*, but *how do we know it* (Aristotle, quoted by Boyer, 51).

We know of Thales and his work only by repute, at second or third hand: nothing of his actual work survives. He travelled to Babylon and Egypt, and was regarded as 'a pupil of the Egyptians and the Chaldaeans' (the Chaldaeans were a Bedouin people who infiltrated Babylonia c. 750 BC; 'Chaldaean' is here used loosely for 'Babylonian'). He is the first individual in history to whom specific mathematical results have been attributed (B 4.2). These include the 'Theorem of Thales': an angle [inscribed] in a semi-circle is a right angle. He is regarded by some scholars (including van der Waerden, but not Neugebauer) as the originator of mathematical proof – crucially important, as proof constitutes the essence of mathematics. Boyer includes two telling quotations: that of Aristotle (above), and '(He) first went to Egypt, and thence introduced this study into Greece. He discovered many properties himself, and instructed his successors in the principles underlying many others' (Proclus, 410-485 AD: Commentary on Euclid Book I).

Thales knew that a piece of amber, when rubbed, will attract small particles of matter. This marks the beginning of electrostatics (study of electricity at rest); the word electricity is derived from the Greek word for amber.[1]

[1]The mathematical study of electricity and magnetism –'EM' – is very interesting, and will be discussed later in chronological order.

Pythagoras of Samos (c. 580 – 500 BC; Samos: Dodecanese island)

Like Thales, Pythagoras[2] travelled to Egypt and Mesopotamia, then returned to found a semi-monastic school – part academy, part secret society – at Croton ('instep of Italy'). His personal influence is difficult to separate from that of the Pythagorean school. The terms philosophy (philo-, loving, + sophistos, wisdom, Greek) and mathematics (mathema: something learned, Greek) are attributed to him. He believed in the transmigration of souls (Shakespeare: *Merchant of Venice* 4.1, 129-132. See also *Twelfth Night*, 4.2, 51-60.).

As such quotations show, he had an impact for beyond mathematics proper. As Boyer puts it (p.57): 'Never before or since has mathematics played so large a part in life and religion as it did among the Pythagoreans'. Perhaps so, but one could argue the converse: the number mysticism (B 4.5) of the Pythagoreans ('All is number') imported into mathematics elements of superstition alien to it and which retarded its progress.

Pythagoras' Theorem: In a right-angled triangle, the square on the hypotenuse [the 'long' side, opposite the right angle] is equal to the sum of the squares on the other two sides.

We have seen strong evidence that this result was known to the Babylonians. Also, Pythagoras is known to have drawn mathematical inspiration from Babylon, and his number-superstition smacks too of Babylonian influence. We do not know whether or not Pythagoras had a *proof* of this theorem. We may have here an early instance of 'Stigler's Law of Eponymy', according to which theorems are rarely, if ever, actually due to the people to whom they are attributed.[3]

The pentagram; the golden section

For a discussion of the 'star pentagram' construction and the 'golden section', see Boyer 4.4, Heath I, 161-2.

Cosmology

The Pythagorean picture of cosmology was one in which the earth and planets (and sun) orbited about a 'central fire'. This sounds very modern nearly two millennia before Copernicus – but alas, it is based on supposed mystical properties of the number ten.

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[2] 'Peetha*γo*ras' in Greek, though 'Pie*thag*oras' is usual in English.

[3] See S. M. Stigler, Stigler's law of eponymy. *Transactions of the New York Academy of Sciences* (2) **39** (1980), 147-157.

Harmonics

Pythagoras is reputed to have noticed that dividing a vibrating string in simple arithmetic ratios produces harmonious overtones. 'Here we have perhaps the earliest quantitative law of acoustics – possibly the oldest of all quantitative physical laws' (B, 63). Again, 'The Pythagoreans were among the earliest people to believe that the operations of nature could be understood through mathematics' (B, 64).

Pythogorean triples.

These are triples of positive integers which can form the sides of a right-angled triangle. The first example one needs it 3,4,5; the next is 5,12,13. There are infinitely many Pythagorean triples, as can be seen from Euclid's formula, that for $m, n \in \mathbb{N}$ with $m > n$, $a := m^2 - n^2$, $b := 2mn$, $c : m^2 + n^2$ is a Pythagorean triple.

The end of the Pythagoreans

After the death of Pythagoras (c. 500 BC) his school continued at Croton, but – because of its conservative political views, and/or its secrecy – was later violently suppressed. The survivors scattered to spread their views in other parts of the Greek world.

Mathematics was carried forward, in this transitional period between the 'founding' by Thales and Pythagoras and the works of the giants of classical Greece whose writing survive, by the scholars considered next.

Anaxagoras (of Klazomenae, NW Peleponnese, d. 428 BC; B 5.2).

'Anaxagoras clearly represented the typical Greek motive – the desire to know' (B p.74). This drive for knowledge for its own sake distinguishes Greek mathematics from the utilitarian concerns of their predecessors (and the Romans), and has remained inseparable from mathematics since.

'Anaxagoras was imprisoned at Athens for impiety in asserting that the sun was not a deity but a huge red-hot stone as big as the Peloponnese'.

According to Plutarch, Anaxagoras in his imprisonment occupied himself trying the *square the circle* – that is, construct a square of area equal to that of a given circle. In the strict sense, 'construct' here means 'by ruler and compass only'. In this form, this famous problem, like the search for the Holy Grail and the Philosopher's Stone (to transmute base metals into gold), obsessed many people and absorbed enormous amounts of effort, before the final proof (1882) that it is insoluble. This is the first recorded mention of this first of the great classical problems, and is a striking example of the pursuit of knowledge for its own sake.

Squaring the circle

'Squaring the circle' has passed into the language as an expression meaning to attempt to do the impossible (or sometimes, loosely, the near-impossible or perhaps-impossible). I have a collection of examples from contemporary journalism; you are encouraged to keep a look-out for your own.

The term is found in literature.[4]

Three famous problems (B 5.3).

In addition to squaring the circle, these were: trisecting an angle, and the 'Delian problem', of duplicating the cube – constructing a cube with volume double that of a given cube, so called because the oracle of Apollo at Delos demanded the doubling of Apollo's cubical altar to avert a plague. All three are impossible. All absorbed much effort, but these unsuccessful efforts did much to fertilise the spectacular progress of Greek mathematics.

Incommensurability (B 5.9)

The Pythagorean dogma that 'all [including geometry!] is number' gave a legitimate place in mathematics of the time only to integers, and to rationals derived from them by division. But, the diagonal of a unit square – an eminently natural geometric entity – may be proved irrational in a few lines by contradiction (B, 83-4). The realisation that irrationals exist, and cannot be denied their place in the centre of mathematics, stunned the Greeks, and precipitated what van der Waerden called the 'foundational crisis'. This discovery may have been made by Hippasus (c. 430 BC); it may have been suggested by the 'golden section' (B 5.10 and fig. 5.6).

There are other views on incommensurability, and on Greek mathematics generally. See e.g. Fowler's book (cited above), and his obituary (Handout). History of Mathematics, like History generally, involves matters of opinion and of interpretation as well as matters of fact.

Aside on the real number system $\mathbb{R}$.

The Greeks were happy with 'real numbers' interpreted as lengths of line segments in geometry. They must have realised that reals may be approxi-

[4]W: The problem of squaring the circle has been mentioned by poets such as Dante and Alexander Pope. The character Meton of Athens in the play The Birds by Aristophanes (first performed in 414 BCE) mentions squaring the circle.
Dante's Paradise canto XXXIII lines 133135 contain the verses:
As the geometer his mind applies/ To square the circle, nor for all his wit/ Finds the right formula, howe'er he tries.
Similarly Alexander Pope, in his 1743 poem Dunciad, and Gilbert and Sullivan, Princess Ida: "And the circle they will square it/Some fine day."

mated by rationals, but since they – unlike their predecessors – *cared* about exact rather than approximate reasoning, this did not reassure them. Of course, the correct path is to *construct* the reals from the rationals, as in Analysis – but this is a genuinely difficult mathematical task, not fully accomplished before 1872.

Instead, the Greeks 'retreated from number into geometry', building a new 'geometric algebra' to replace the 'arithmetic algebra' they thought – understandably but wrongly – discredited by the incommensurability crisis. Artificial though this may seem – and unnecessary as it certainly was – it still has a persuasive pictorial impact (figures to illustrate this on the board).

Zeno (of Elea, fl. c. 450 BC) (B 5.11).

Zeno introduced certain 'paradoxes' (Achilles and the tortoise is the most famous example), highlighting (in modern language) the continuity rather than the discreteness of the reals. As the Greeks were unable – without the reals – to handle limiting processes to their own high standards of logic and rigour, these deepened the foundational crisis. [Aside: look no further for the justification of Analysis in the Mathematics curriculum of today!]

Democritus (of Abdera, born c. 460 BC) (B 5.14).

Archimedes attributed to him the formula

$$V = \frac{1}{3}Ah$$

for the volume of a circular cone. Rigorous proof of this requires ideas of calculus, which lay far beyond Democritus, but here we have a first inkling of 'pre-calculus' in Greek mathematics.

Perspective in the Ancient World

We know from the writings of classical authors, such as the Roman author Vitruvius, and from surviving wall- and vase-paintings, that some elements of perspective were known to the ancient Greeks. Agatharchos used it for stage sets in the late 5th C. BC. Theoretical studies of perspective were made by Anaxagoras (above, and B 5.2) and Democritus (above, and B 5.14).
W: Systematic attempts to evolve a system of perspective are usually considered to have begun around the fifth century BC. in the art of Ancient Greece, as part of a developing interest in illusionism allied to theatrical scenery and detailed within Aristotle's Poetics as 'skenographia'.

THE AGE OF PLATO AND ARISTOTLE (B Ch. 6)

While we have no mathematical texts from this time, we do have distories and drama, and the writings of philosophers. In those days, human knowledge was much less extensive than today, and leading thinkers were able to keep abreast of progress in fields other than their own.[5]

Socrates (C. 469 – 399 BC) (B 6.2)

Socrates was an Athenian philosopher, who fought in the Peloponnesian War. He is remembered for Socratic ignorance (a technique of argument in which one uses pretended ignorance to confuse an opponent), and for 'questioning everything on principle'. This was thought by the Athenian elders to be a dangerous influence on the young, and he was forced to kill himself (by drinking hemlock). Socrates did not value mathematics highly.

Plato (Roman form: Platon in Greek, hence 'Platonic') (428/7 – 348/7 BC) (B 6.3)

Plato was a pupil of Socrates, and like him was primarily a philosopher. His philosophical works include Plato's Republic, Dialogues etc., and writings on Socrates.

Although Plato himself produced no mathematics of lasting importance, he did become the mathematical inspiration of the 4th C. BC. Plato's Academy in Athens was the first institution of higher learning in the world (unless one counts Pythagoras' school, which was in part a secret society), and was the principal mathematical centre of its time. Over its door was inscribed the motto 'Let no one ignorant of geometry enter here.'[6]

The Platonic solids

How many solid figures are there whose faces are congruent regular polygons?

[5] The days have long passed when it is possible to keep abreast of one's own subject as a whole. Learning has thus led to a degree of fragmentation. No member of this or any other Mathematics Department can claim to have a thorough knowledge of the subject across the board. One advantage of History of Mathematics as a subject is that it serves as a unifying influence against this tendency to fragmentation.

[6] A mark of the enduring impact of Plato's Academy is that a drawing of it, with this motto, is used in the logo of the American Mathematical Society (AMS), which has for some 70 years been the pre-eminent mathematical society in the world.

Triangular faces. How many faces (equilateral triangles) meet at a vertex? If 3, we have a *tetrahedron*; if 4, an *octahedron*; if 5, an *icosahedron* (2 or less would not give a solid; 6 gives instead a triangular tessellation of the plane, again not a solid; there is no room for 7 or more).
Square faces: 3 gives a cube (4 gives a square tessellation of the plane – 'OS grid').
Pentagonal faces. 3 gives a dodecahedron (4 is impossible, as the angle at the vertex of pentagon is $3\pi/5$ and $4 \times 3\pi/5 > 2\pi$).
Hexagonal faces give a honeycomb tessellation of the plane, and not a solid. So the list above is exhaustive.

These 5 'regular polyhedra' are called the *Platonic solids*, after Plato's use of them in the Dialogue of Timaeus. Apparently, according to a scholium to Book XIII of Euclid's Elements, the tetrahedron, cube and dodecahedron were known to the Pythagoreans, while Theaetetus (d. 369 BC) found the octahedron and icosahedron (he probably also proved the theorem above: the list is exhaustive). This is perhaps surprising: it is the dodecahedron and icosahedron that seem the hardest, while the octahedron is merely a pyramid (!) reflected in its base-plane. But archaeological support exists: an Etruscan dodecahedron made c. 500 BC of stone was found in 1885 on Monte Loffa near Padua, and a number of Celtic examples have also been found. Perhaps also the idea of reflection is not as self-evident as it may seem today.

For background, see e.g.
Benno ARTMANN: Roman dodecahedra. *Mathematical Intelligencer* 15 (1993), 52-53. [7]

Unfortunately, Plato viewed the 5 solids as endowed with mystical significnace, a throwback to the superstition of the Pythagoreans and the Babylonians before them (earth: cube; air: octahedron; fire: tetrahedron; water: icosahedron; universe: dodecahedron).
Natural occurrence
Crystals: Tetrahedron: sodium sulphantimoniate; cube: common salt; octahedron: chrome alum
Living forms: skeletons of radiolaria (microscopic sea creatures):
Circogonia: icosahedra; Circorrhegma: dodecahedra.

[7] For anyone interested, and who can read German, I possess
Eva SACHS, *Die fünf Platonischen Körper: Zur Gesichte der Mathematik und der Elementenlehre Platons und der Pythagoreer*, Weidmannsche Buchhandlung, Berlin, 1917.

Eudoxus of Cnidus (c. 408 – c. 335 BC) (B 6.7-9)

'Eudoxus was without doubt the most capable mathematician of the Hellenic Age' (B, 102). None of his actual writings have survived, but his main contributions are known to be:

1. *Theory of Proportion*

Two numbers are *commensurable* if their ratio is rational, incommensurable otherwise. The crisis over irrationals was also a crisis over incommensurables, hence over ratios. Eudoxus introduced a rigorous treatment of ratios, or proportions, based on the definition of equality of two ratios:

$$a/b = c/d$$

iff (combining '$<, =, >$' statements together as '$< / = / >$' for convenience)

$$\forall m, n \in \mathbb{N}, \qquad ma > / = / < nb \quad \Rightarrow mc > / = / < nd.$$

This symbolic summary of Eudoxus' definition of equality of propertions resembles cross-multiplication:

$$a/b = c/d \qquad \text{iff} \qquad ad = bc.$$

As 'proportion' is to numbers, so 'similar' is to figures in geometry. We shall return to similarity and proportion in discussing Euclid's Elements. Note however that the Greeks were reluctant to use division, or ratios, which affected the ordering of material in Euclid.

2. *Method of Exhaustion*

The 'lemma of Archimedes', or 'Archimedean property of the reals' (apparently actually due to Eudoxus) simply says (in modern language) that

$$\forall \epsilon > 0 \qquad \exists N \in \mathbb{N} \quad \text{such that} \quad 1/N < \epsilon,$$

or more succinctly $1/n \to 0$ as $n \to \infty$; similarly $r^n \to 0$ as $n \to \infty$.[8]

Eudoxus based in this his 'method of exhaustion', essentially a primitive form of the integral calculus. Archimedes attributes to Eudoxus the first rigorous derivation of the volume of a cone, and perhaps of the area of a circle and the volume of a sphere.

[8] In Field Theory, a branch of Modern Algebra which we discuss in chronological order later, such fields are called *Archimedean fields*.

Note. The area of a circle may be approached by inscribing and circumscribing an n-gon (regular polygon with n sides) in a circle and letting $n \to \infty$.
3. *Astronomy* (B 6.9; Dreyer Ch. IV).

Eudoxus is regarded as the 'father of scientific astronomy', for his theory of 'homocentric spheres' for the orbits of the sun, moon and five (then) known planets.

Menaechmus (fl. c. 350 BC) (B 6.10,11, Coolidge Ch. I).

Menaechmus was a pupil of Eudoxus. In an attack on the Delian problem, he discovered the *conic sections*, or *conics* (the ellipse, parabola and hyperbola, with the line, line-pair and point as special or limiting cases) – the locus of intersection of a right circular cone by a plane.

Menaechmus taught Alexander the Great, and is reputed to have told him, when asked for a short cut, that 'there is no royal road to geometry' (a story repeated later with Euclid and Ptolemy).

Aristotle (c. 384-322 BC)

Aristotle was the foremost scholar of is age, a polymath (though not himself an eminent mathematician) and tutor of Alexander the Great. On Alexander's death in 323 BC Aristotle lost influence and left Athens; he died the next year.

The Hellenic and Hellenistic Periods

From its foundation c. 330 BC by Alexander the Great, Alexandria in Egypt began to replace Athens as a centre of learning. It is customary to classify Greek history as *Hellenic* before this time, *Hellenistic* or Alexandrian after it, and we shall follow this.

Euclid (of Alexandria; fl. c. 300 BC)[9]

Ptolemy I succeeded Alexander the Great in Egypt, and consolidated his rule by 306 BC. He established the Museum at Alexandria, the leading academy (in effect, university) of its day. Here Euclid taught, and wrote, around 300 BC, the most successful mathematics textbook ever written, the *Elements*. He also wrote the *Optics*, the *Phaenomea* (on spherical geometry), *Division of Figures* and other works, such as the *Surface Loci*, now lost.

Euclid's great gift was exposition: the Elements is a *textbook* on elemen-

[9]'Ef*klid*is' in Greek

tary mathematics. Thus it excludes logistic (as too mundane), and conic sections (as too advanced), but includes arithmetic (= number theory), geometry and geometric algebra.

As today, extensive references and attribution of credit is appropriate for research but not for elementary exposition. Thus Euclid does not assign credit, and so it is difficult for us to know how original his work was. It was by no means the first text of its kind. However, it has lasted to this day, while other texts did not; this and the great impact the Elements had suggest that it was the best textbook of its kind. The Elements is divided into thirteen books: Books I-VI on elementary plane geometry, VII-IX on the theory of numbers, X on incommensurables, XI-XIII on solid geometry.

Euclid (we speak of the author and his Elements interchangeably from now on) expounds mathematics – particularly geometry – in what is essentially the modern style – *axiomatically.* One begins with a list of *axioms* (as few as possible, and as 'obvious', or 'innocuous', as possible). From these, one deduces, by rigorous proof, *theorems* (as many as possible, as 'non-obvious' as possible).

The Axioms.

Take the terms 'point' and 'line' as basic and undefined, and take the context as the plane (also undefined!). Axioms 1-5 are:

A1. Two (distinct) points determine a (unique) line.

A2. A line-segment can be extended ('produced') to give a line.

A3. Given any point and any radius, there is a circle with that point as centre and that radius.

A4. All right-angles are equal.

A5 (Parallel Postulate). If line L makes angles θ_i with lines L_i $(i = 1, 2)$, and $\theta_1 + \theta_2 < \pi$, then L_i produced intersect at a point P on the same side of L as angles θ_i (non-parallel lines intersect).

Book I (Propositions 1-48):elementary geometry of triangles, parallelograms etc. Pythagoras' theorem is Proposition 47.

Book II (Prop. 1-14): geometric algebra. For example, Proposition 10 is the geometric counterpart of the algebraic identity $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$.

Book III (Prop. 1-27): geometry of the circle. E.g.:

Proposition 21: Angles in the same segment of a circle are equal.

Proposition 22: Opposite angles of a cyclic quadrilateral sum to π.

Proposition 31: Theorem of Thales.

Book IV (Prop. 1-16): inscribed and circumscribed polygons in a circle.

Boook V (Prop. 1-25): theory of proportions (cf. Eudoxus).

Book VI (Prop. 1-33): applications of proportions; similar triangles, etc.
Book VII (Prop. 1-39): elementary number theory. Proposition 2: hcf, by 'Euclidean algorithm'; Proposition 24: a and b coprime to n implies ab coprime to n; Proposition 34: lcm.
Book VIII (Prop. 1-27): geometric progression; squares and cubes, etc.
Book IX (Prop. 1-36): primes. Proposition 20: There are infinitely many primes. Proof: by contradiction.
Book X (Prop. 1-115): incommensurability; surds. Proposition 17: commensurability properties of roots of quadratics.
Book XI (Prop. 1-39): solid geometry. Planes, parallelepipeds, etc. Proposition 3: two planes that meet do so in line.
Book XII (Prop. 1-18): areas and volumes. Proposition 10: $V = \frac{1}{3}Ah$ for a cone, by the 'method of exhaustion'.
Book XIII (Prop. 1-18): the Platonic solids. Propositions 13-18: lengths of the edges as a function of the radius r of the circumscribing sphere.
Books XIV and XV are apocryphal (XIV perhaps by Hypsicles, XV probably by Miletus); see Heath for details.
The Parallel Postulate. Axioms I-IV are as innocuous as any axioms could be, but Axiom V – the Parallel Postulate – is different. Enormous efforts were made to prove it from the other axioms. These efforts were doomed to fail, as was shown some 21 centuries later by the *non-Euclidean geometries* of Bolyai and Lobachevskii in 1829.
Optics and *Catoptrica* (= Theory of Mirrors).

The Optics summarises Greek knowledge of perspective, and the Catoptrica (perhaps not by Euclid himself) includes the law of reflection of light at a mirror: angle of incidence = angle of reflection.

Overall assessment

It may be interesting to compare two very different views of Euclid by two distinguished 20th C. mathematicians.
Albert Einstein (1879-1955): 'If Euclid failed to kindle your youthful enthusiasm, then you were not born to be a scientific thinker' (quoted in *American Math. Monthly* **99** (1992), 773).
Salomon Bochner (1889-1982): 'In short, it is almost impossible to refute the assertion that the Elements is the work of an insufferable pedant and martinet' (from *The role of mathematics in the rise of science*, PUP, 1966, 1.7, p.35).