m3hw1.tex **Week 1. 17 – 20.1.2017**

## PRE-HISTORY TO GREEK HISTORY

*Timeline*

Ice Age, ended (in Britain) c. 13,000 years ago

Stone Age (lithos = stone, Greek):

early (paleolithic);

Lower Paleolithic, c. 2.6 million – 300 thousand years ago;

Middle Paleolithic, 300k – 30k;

Upper Paleolithic, 50k – 10k;

middle (mesolithic);

late (neolithic).

The neolithic age is taken as roughly 4,500 BC – 2,000 BC in W. Europe, though it began much earlier in the Middle East (c. 9,000 BC).

Bronze Age (bronze is an alloy of copper and tin):

Early Bronze Age [EBA], 3,300 – 2,100 BC;

Middle Bronze Age [MBA], 2,100 – 1,550 BC;

Late Bronze Age [LBA], 1,550 – 1,200 BC.

Iron Age

Early Iron Age [EIA], 1,300 – 500 BC;

Middle Iron Age [MIA], 500 BC – 250 AD;

Late Iron Age [LIA], 250 – 500 AD.

*Dramatis Personae*

Mitochondrial Eve, c. 200,000 years ago.

*Pre-history: technology*

Archaeologists traditionally divide human pre-history by its technology, divided into the Stone Age, Bronze Age and Iron Age, in that order. While mathematics proper is Iron Age, "pre-mathematics" can certainly be found in the Bronze Age. From Wikipedia [W]: "The Bronze Age in the ancient Near East began with the rise of Sumer in the 4th millennium BCE. Cultures in the ancient Near East (often called, "the cradle of civilization") practised intensive year-round agriculture, developed a writing system, invented the potter's wheel, created a centralized government, law codes, and empires, and introduced social stratification, slavery, and organized warfare. Societies in the region laid the foundations for astronomy and mathematics."

*Pre-history: evolution*

Mathematics is a human construct. While mathematics proper is only

about two and a half thousand years old, humanity stretches back in time for much longer. Our species, homo sapiens, is descended from apes (the chimpanzee and orang outang are among our closest relatives). All living humans are believed to be descended from one woman, known as Mitochondrial Eve, c. 190,000-200,000 years ago, [1][2] in Africa. Homo sapiens subsequently expanded from Africa to colonise the world.

Early human societies were communities of hunter-gatherers – nomads who lived off the land. This state of affairs lasted for a very long time. It did not produce any mathematics or science that need concern us, but this period did see the evolution of language, a pre-requisite for everything else that we discuss.

*Pre-history: language*

While animals generally communicate with each other, and with us, very well, language proper is specific to our species, Homo sapiens. There is debate as to whether language evolved incrementally over a long period of time, or comparatively suddenly in a "Great Leap Forward" (some scholars call this the Upper Paleolithic revolution, and date it to c. 100k years ago). Language includes the ability to communicate information without restriction on length or complexity. Language is a necessary preliminary for everything that we shall discuss later.

The study of early language, or languages, is complicated by the absence of written records. Nevertheless, language specialists have been able to deduce the "family tree", or line of descent, of the languages to be found today (and to some extent, of now extinct ones). For example, English (a comparatively recent language, which evolved from Anglo-Saxon c. 500 AD, with additions from Norman French from 1066, through Middle English of Chaucer's time) is a member of the Indo-European family of languages, which originate from Sanskrit[3]. Also relevant to us are Chinese, and the Semitic languages (Hebrew, Arabic, etc.).

*Pre-history: art.*

The last great Ice Age (which saw much of what is now Britain buried

[1] See e.g. W, Mitochondrial Eve.

[2] Mitochondria are found in every cell of the body, and organise the release of energy to power the cell. While our DNA is derived 50-50 from our father and mother, our mitochondria come entirely from the mother. Mitochondrial Eve is so called because she had an unbroken line of *female* descendants.

[3] Sanskrit evolved in India. The early form, Vedic Sanskrit, dates back to 1,500 – 1,200 BC. Panini, (4th C. BC) wrote the classic Sanskrit grammar.

beneath deep glaciers) ended c. 13,000 years ago. We have considerable archaeological evidence of human activity from the Ice Age, including cave paintings, often of wild animals, e.g. in the caves of Lascaux, France. Works of art from this period were featured in a recent exhibition at the British Museum (BM from now on)[4]

*Agriculture*

It is generally agreed that agriculture (rather than hunting or nomadic tending of herds) is a necessary prerequisite for the emergence of a society (or civilisation – below) complex enough to support interesting (or indeed, non-trivial) science or mathematics, and that early (neolithic) agriculture has been practised for some 10 millennia.

The seeds of some varieties of grass are nutritious. It was observed that such seeds can be gathered, ground up (into flour) and cooked (into bread); furthermore, that a portion of the harvest can be retained and used as seed-corn to generate the next harvest, etc.

Wine-making can be traced back to Georgia, c. 6000 BC.

*Civilisation*

The first major civilisations relevant to early mathematics were potamic (river-based: potamos = river, Greek), in the Fertile Crescent (Mesopotamia, between the rivers Tigris and Euphrates, in modern Iraq), Egypt (Nile), China (Yangtse), India (Indus). Each of these produced mathematics worth studying, and we shall do so below.

*Counting*

Learning to count is the first mathematical experience of each individual, and of each society. At this point, please pause to search your own memories here. Please also resolve to observe this experience in your own children (and pupils, if you go into teaching). Anthropology contains examples of cultures with a counting system of "one, two, many", but non-trivial counting goes back at least 30 millennia (Boyer, p.4, describes a notched bone from this period). A. Seidenberg (1960s) suggested a ritual origin for some prehistoric counting systems.

It would be too achronological to talk here about different *bases* for counting systems; we return to this later.

*Writing.*

Ancient writing is important to us for several reasons, e.g.

[4]Ice Age Art: Arrival of the modern mind, BM, 7.2 – 26.5.2013. Catalogue: Jill Crook, *The swimming reindeer*, British Museum Press, 2010.

(a) indicating a level of developed civilisation;
(b) providing a language in which ideas – mathematical or otherwise – can be communicated (to contemporaries of the writer, and if permanent, to us);
(c) providing mathematical texts proper.
We shall return to this in the contexts of Egypt, Mesopotamia, India and China.

*Astronomy*

Study of the Sun and Moon has religious roots ("Sunday", "Monday", ...) predating civilisation. An ability to predict seasonal change accurately is essential to successful agricullture, and this necessitated early study of astronomy, calendars, observatories etc. There is some reason to date the invention of the solar calenar in Egypt c. 4,241 – 4,228 BC (Boyer p.14); we discuss early observatories below.

*Coinage/money*

In primitive societies, trade was done by barter, and wealth was accumulated in goods, e.g. cattle (and, in agricultural societies, land). For instance, our words 'pecuniary' and 'impecunious' are derived from the Latin 'pecunia', itself derived from 'pecus' = cattle, and in some African societies wives must still be purchased from their families on marriage with cattle. Again, our 'salary' is derived from sal = salt: Roman soldiers were at one time paid in (or had their pay reckoned in) salt (hence also the phrase 'worth his salt').

As agriculture, civilisation, urbanisation and trade developed, the need for money as a medium of exchange developed with them. The earliest coins known date from c. 700 BC in Lydia (Asia Minor, modern Turkey), made of electrum (pale gold, containing silver); some are in the BM. The system of weights used is Babylonian.

The study of coins, or numismatology (numisma = current coin, Greek) can tell us much about early societies (counting systems, degree of social and political organisation, etc.). Note that early British coinage pre-dates the Roman conquest.

*Megalithic monuments* (mega = great; lithos = stone).

The best-known British examples are Stonehenge (I, c. 2,800 BC; II, c. 2,150 BC; IIIabc, to c. 1,550 BC), and the nearby Avebury Ring, and Newgrange, Ireland (c. 3,000 BC). These monumental structures are clearly (in part) neolithic observatories. Scholars disagree on their precise functions, but they are clearly highly informative about the societies that built them – e.g., the bluestones in Stonehenge II (up to 5 tons) have been identified geologically as from the Mynydd Preseli, Pembrokeshire, 250 miles away.

**EGYPT**

*Timeline*

The Thirty Dynasties, c. 3,200 – C. 300 BC

The Old Kingdom, Third – Sixth Dynasties, c. 2,800– c. 2,300 BC

The Pyramid Age, c. 2,800 – c. 2,500 BC

*Civilisation*

Civilisation may be regarded as a prerequisite for the doing of serious mathematics. The dominant form of early civilisation was empire (indeed, this form predominated until after World War I!). Empires need to raise taxes to pay for armies, administration and the law. This requires the ability to write, and to calculate.

Egyptian civilisation was based on the Nile, and in particular, the regularity of its annual flood (on which their agricultural system was based), around June 15. The early Egyptians learned both the 365 days of the year and the 1/4 day ('leap-year') first-order correction, giving a 'cycle' of c. 1,460 years[5]. We know (Censorinus, *De die natale*, 238 AD) that the calendar was correctly aligned in 139 AD. Backward extrapolaton through two cycles suggests a time-origin c. 2,781 BC (or 2,773 BC with correction), by which time pyramid-building was beginning. If one regards the pyramids as evidence of long pre-existing civilisation, a three-cycle hypothesis gives time-origin c, 4,228 – 4,241 BC.

*Writing*

Egyptian writing originated as hieroglyphics (Greek: 'sacred carvng') before 3,000 BC. The original pictographic form, used for carving, was too complicated for everyday use, and was successively simplified to hieratic ('priestly') and demotic ('popular': demos = people, Greek). Our knowledge of these owes much to the *Rosetta Stone*, a trilingual inscription (in hieratic, demotic and Greek) recording a decree commemorating the coronation of Ptolemy V in 196 BC.[6] Discovered by Napolean's army in Egypt, it was captured by the British, and is now in the BM (ground floor, 'left and right' – not far from the Elgin Marbles).

Papyrus, a plant growing in the Nile valley, was used to make paper.

*Numerals*

Hieroglyphic numerals (discussed by Boyer, 2.2) go back at least 5 mil-

[5]The $365\frac{1}{4}$ day year was imported from Alexandria to Europe by Julius Caesar – his greatest legacy to mathematics and science, and indeed to our lives generally.

[6]Ptolemy was one of the generals of Alexander the Great (356-323 BC), and ruled Egypt, including Alexandria (founded by Alexander c. 331 BC) after Alexander's death.

lennia. They used a scale of ten, and by suitable combination of a small number of standard symbols, large integers could be compactly represented. Thus in principle (though less convenient in practice) the Egyptian system had some essential features of modern decimal notation.

*Pyramids*

Apart from the Nile, the other main factor in Egyptian thinking seems to have been their preoccupation with death, the after-life, etc. During the period c. 2,800 – c. 2,500 BC, the pyramids were erected, presumably as royal graves, at vast human and material cost. The Great Pyramid of Khufu (or Cheops), c. 2,750 BC, is 450 feet high and contains 75 million cubic feet of masonry. According to Herodotus (Greek historian, 5th C. BC), 300,000 mean spent 10 years quarrying the stone and 10 years building it.

*Geometry*

The word geometry means 'earth-measuring' (Greek). The high standards of accuracy in the building of the Pyramids indicates that their architects and surveyors ('rope-stretchers') had great practical efficiency in measurement and design.

The perimeter:height ratio of the Great Pyramid is close to 44/7. There has been some debate as to Egyptian estimates of $\pi$ based on this, but K. Mendelssohn (1974) points out that the rope-stretchers could have achieved this by a 4:1 rule of thumb and uncoiling ropes from a circular drum

*Papyrus texts*

*RMP: Rhind, or Ahmes, Mathematical Papyrus* (in BM; purchased by H. Rhind in 1858; coped by the scribe Ahmes c. 1,650 BC).

*MMP: Moscow Mathematical Papyrus* (c. 1,890 BC).

Mathematical contents include:

a. Use of cipher numerals – a contracted form of hieroglyphic numerals;

b. Use of addition and multiplication;

c. Algebra: solution of linear equations;

d. An approximation to $\pi$: $\pi \sim 256/81 = 3.1604....$

e. Volume calculations, e.g. $V = Ah/3$ ($V$ = volume, $A$ = base area, $h$ = height of tetrahedron, pyramid etc.). The volume of a frustrum (truncation) of a pyramid was also calculated.

f. 'Egyptian' or unit fractions. The Egyptians were obsessed with unit numerators. Thus they used $1/4 + 1/5$ in preference to $9/20$, $1/10 + 1/30$ for $2/15$, etc. This absorbed much of their mathematical energies, and undoubtedly had a stultifying effect on Egyptian mathematics generally.

The non-unit fraction $2/3$ had special status and a special symbol.

## MESOPOTAMIA

*Chronology*

Our knowledge of Mesopotamian pre-history goes back to the Sumerian (cf. Akkadian) period, c. 4,000 – c. 1,800 BC (Sumer and Akkad were Mesopotamian place-names). The Sumerians were non-Semitic, and may have been linked to India. Later the Babylonians (and Assyrians, whose capital was Nineveh rather than Babylon) took over the region, and assimilated much of Sumerian culture. The Babylonian Empire lasted from 1,894 – 539 BC, when Cyrus of Persian conquered the area. Greek influence came later with conquest by Alexander the Great (356-323 BC). He was succeeded in Mesopotamia by his general Seleucus; the term Seleucid is applied to this period. The term Babylonian is often used loosely for Mesopotamian.

Mesopotamian culture was highly developed, and many of its aspects leave traces surviving to this day.

*Writing*

The Sumerians wrote on clay with a stylus, leaving wedge-shaped marks which were baked hard in the sun on clay tablets. This cuneiform script (cuneus = wedge, Latin) originated during the 4th millennium BC (as did use of the wheel). Clay tablets were more durable than papyrus, so more Mesopotamian than Egyptian writing has survived.

*Civilisation*

Mesopotamian culture excelled in military technology (early use of the horse, chariot, phalanx, etc.), in law-making (the bulk of surviving tablets relate to legal matters) and building (e.g. ziggurats or staged towers – hence our word zigzag)[7], the Hanging Gardens of Babylon, etc.

*Myths*

The Sumerians had myths of:

creation of the world in 7 days (hence the origin of our modern 7-day week, and the 'magic' status of the number 7); cf. the Hebrew creation story of the Bible;

a great flood (the Tigris and the Euphrates were more erratic than the Nile); again, cf. the Noah story of the Bible.[8]

[7] A modern example is the Business School of Oxford University, near Oxford station

[8] According to the Black Sea deluge hypothesis, massive flooding occurred from the Aegean (the part of the Mediterranean between Greece and Turkey) and the Black Sea c. 5,600 BC. There are other theories (see W for details), but clearly some such flooding did occur.

The Babylonians developed astrology; in particular the signs of the zodiac stem from them (hence also our word disaster: dis + astrum = star, Latin: disastrous = ill-starred).[9]

*Counting*

The base of Mesopotamian counting was 60 (chosen presumably for ease of division by 2,3,5 etc.), and there are similarities between cuneiform and heiroglyphic numerals. The Mesopotamian system was superior in that
a) it used *positional notation*, in the manner of our decimal notation with base 60 in place of 10 (B, 3.2);
b) it handled *fractions* as flexibly as integers – as our notation for decimals (= decimal fractions) does today – a crucial advantage over the clumsy Egyptian unit fractions (B, 3.3). They could e.g. extract square roots (they found $\sqrt{2}$ accurate to 1 in $10^5$), and they constructed mathematical tables (B, 3.4).

The base 60 survives today in the minutes and seconds used to measure time and angle.[10] It also presumably inspired our $360^o$ circle (B 10.4).

*Algebra*

The Arabs (considered later) were traditionally regarded as the originators of algebra, but Otto Neugebauer (1930 on, in his book of 1951/57) showed that Mesopotamian texts some 4,000 years old showed familiarity with solutions of (some) *quadratic equations*. They even considered some *cubic* and *quartic equations* (B 3.7) – an astonishing achievement that anticipated the final solution of such problems by three and a half millennia.

*Geometry*

The Plimpton 322 tablet (c. 1,900 – 1,600 BC) contains a list of Pythagorean triples; ratios of corresponding areas are calculated to 8 sexagesimal (14 decimal) places (B 3.8). Areas of regular polygons of 3,4,5,6 and 7 sides were calculated, as was the approximation $\pi \sim 3\frac{1}{8}$ (Susa tablets: B 3.9). They knew the diagonal ($\sqrt{2}$) of the unit square to within a millionth, implying (apart from great computational skill) a working knowledge of the 'Pythagorean' theorem (Yale tablet: B 3.10). However, like other ancient civilisations, they failed to distinguish the exact from the approximate. The extent to which Mesopotamian (or other ancient) mathematics had any capacity for abstraction, or concept of proof, is a matter for scholarly debate.

[9] Astrology is obvious nonsense, yet large numbers of people still read astrology columns in newspapers, and a number of quality newspapers, which should know better, carry them.

[10] These terms are contractions of the Latin 'partes minutae primae' and 'partes minutae secundae' – small parts of the first and second kinds; cf. our adjective minute = very small.

**THE GREEKS: HISTORY**

*Timeline*

Minoan civilisation, Crete, 27th – 15th C BC [11]

Mycenean civilisation, N. Peloponnese, c. 1600 – c. 1100 BC [12]

Linear B (Knossos, Crete), c. 1400 BC (first written evidence); deciphered by Michael Ventris (1922-1956)

Trojan War, c. 1,250 BC

Homer, author of the *Iliad* (account of the Trojan War) and *Odyssey* (of the journey home of Odysseus/Ullysses), 8th/7th C BC

Founding of Carthage by the Phoenicians, 800 BC

First Olympic Games, 776 BC

Founding of Rome, 753 BC

First Persian invasion of Greece: battles of Marathon and Artemisium (naval), 490 BC; second Persian invasion of Greece: battles of Thermopylae and Salamis (naval), 480 BC

Herodotus ("father of history"), *Histories*, 450s – 420s BC

Peloponnesian War, 431-404 BC, won by Sparta against Athens

Founding of Alexandria c. 330 BC (Alexander the Great, 356-323 BC)

First Punic War 264-241 BC, Second Punic War 218-201 BC, Third Punic War 149-146 BC; obliteration of Carthage (poeni = Cathaginians, Latin)

Greek peninsula comes under Roman rule, Battle of Corinth, 146 BC

Athens revolts; crushed by Sulla, 88 BC

Roman Egypt (and Alexandria), 30 BC: Octavian (later Augustus) defeats Mark Antony and Queen Cleopatra at the Battle of Actium, and annexes the Ptolemaic kingdom of Egypt to the Roman Empire

Augustus (first Emperor) sets up Greek province of Achaea, 27 BC

Constantine founds Byzantium as capital of the Roman Empire (in the East), 330 AD

Christianisation, 4th – 6th C AD; Justinian I closes the Academy of Athens, 529 AD

*Periods*

Archaic Period (c. 800 – c. 500 BC); Athenian democracy 508 BC

[11] Archaeological evidence points to a great volcanic eruption on the island of Thera [Santorini], N. of Crete, c. 1628 BC, and great tsunamis destroying N. Crete. This is believed to account for Plato's description of the lost city of Atlantis.

[12] Pelops, a hero of Greek mythology; Peloponnesos = Island of Pelops, the S. Greek peninsula

Classical Period (c. 500 - 323 BC – death of Alexander)
Hellenistic Period (323 – 146 BC – Battle of Corinth)
Roman-Greek Period (146 –)

*Background reading*

H. D. F. KITTO, *The Greeks*, Pelican 1951/Penguin 1991
C. M. BOWRA, *The Greek experience*, Weidenfeld & Nicholson 1957
Donald KAGAN, *The Peloponnesian War*, HarperCollins 2003
Tom HOLLAND, *Persian Fire: The First World Empire and the Battle for the West*, Abacus 2005
David ABULAFIA, *The Great Sea: A Human history of the Mediterranean*, Penguin 2012

The Greek contribution to mathematics is so spectacular that we shall need to study it at length. The Greeks originally entered what is now Greece by migrating southwards from NE Europe, so far in antiquity that we have no written records.

*Language*

Greek has been spoken in and around what is now Greece for at least two millennia BC (plus two AD). This makes it the oldest recorded living language. Among the Indo-European group of languages, its age matches that of now-extinct languages that we know of.

As noted above, the first written evidence of Greek is Linear B (Knossos, Crete, c. 1400 BC). This was deciphered by Michael Ventris (1922-1956). [13]

Koine Greek was a fusion of ancient Greek dialects with Attic, the form of Greek spoken in Athens. This became the lingua franca (common language) of the E. Mediterranean and Near East. The Bible was first written in Greek (biblos = book, Greek) (the New Testament was first written in Greek; the Old Testament, the holy text of Judaism, was written in Hebrew and translated into Greek).

*Writing*

Greek script developed from a vowel-less script (possibly Phoenician, derived from cuneiform and/or hieratic) by addition of vowels. Note that of the Semitic scripts, Hebrew is still wholly and Arabic largely vowel-less. Hebrew and Arabic are still written *from right to left*. Etruscan (N. of Rome, modern Tuscany; Etruscan kings ruled early Rome) and Oscan (S. of Rome) were also written right to left.

[13] There is a blue plaque commemorating Ventris and his achievement in Hampstead (Wildwood Terrace, off North End, behind The Old Bull and Bush pub).

The Greek alphabet emerged during the 8th C. BC, with 24 letters, alpha, beta [hence alphabet] to omega.[14] The Roman alphabet developed from the Greek alphabet, during the 7th C. BC, via Cumae, a Greek colony in [what is now southern] Italy. Both the Roman and Greek alphabets, as they developed, were and are written *left to right*.

*Numerals*

The older, Attic, system of numeration – rather similar to the later Roman numerals – gradually gave way to the Ionic or alphabetic system (B 4.10). This notation is quite effective for integers, but less so for fractions, where the Egyptian preference for unit fractions lingered.

*Economics*

The Greeks traditionally excelled as merchants, and as sailors. Greek influence, and the Greek language, spread from the Greek peninsula across the Aegean to its East to Asia Minor, the Adriatic and Ionian Seas to its West to what is now southern Italy and to Sicily, and the Mediterranean to its South and SE to Egypt and the W. coast of the Middle East.

*Politics*

By the time of which we have records, Greece was a collection of city states (each formed a polis, or community, hence our word politics). Over time, city states founded colonies in other parts of the Hellenistic world. It was jealousy and squabbling between parent cities and their colonies, as these grew more self-assertive, that laid the foundations for the appallingly bloody and destructive Peloponnesian War. By that time, the dominant city states were Athens (called after the goddess Athena) and Sparta; Corinth and Thebes were also important.

Athens is noted as the birthplace of democracy (demos = people; kratos = rule) – but Athens had many slaves (and of course women did not have the vote). Athens led the Delian League (named after Delos, the island where it was set up), but this evolved into a euphemism for the Athenian Empire.

Sparta was a militaristic state, where the men were trained above all to fight. The agricultural labour was done by helots – the inhabitants of a nearby area that Sparta had conquered. Sparta lived in fear of a successful uprising of the helots. It was the Spartan Three Hundred, under their King Leonidas, who died defending Greece against the Second Persian Invasion.[15]

[14]The Greek alphabet will be familiar to those who speak modern Greek, and those who studied classical Greek. The rest of us in mathematics have a partial familiarity, but it is worth learning the Greek alphabet in full.

[15]W: A Greek force of approximately 7,000 men marched north to block the pass in the

Macedonia, to the north, was a kingdom – a form of government of which the Greeks disapproved. King Philip II of Macedon built up a powerful army, which (after he was assassinated in 336 BC) was used by his son Alexander to conquer, not just Greece, but Asia Minor, the Persian Empire, and much of the known world, including (parts of) Afghanistan and India.[16]

The conquest of Greece by Alexander cleared away the old city states. When his empire was divided up after his death, this left a power vacuum. This was filled by the aggressively expanding new city state of Rome, as it built up the Roman Empire, under the Republic during the period that concerns us, and the Emperors later.

*Cultural influence*

Greece excelled in (as well as introducing democracy, however imperfectly) philosophy and mathematics. Rome excelled in warfare (military at first, naval also later), politics (neighbouring countries were encouraged (or coerced, as the case might be) to accept Roman protection, and place themselves *in fidem populi Romani* (in the trust of the Roman people)), engineering (Rome built excellent roads – almost all the straight roads in the UK are of Roman origin; many Roman buildings are still standing)[17]. Greek learning enjoyed high prestige among the ruling Roman elite, many of whom spoke Greek in preference to Latin. This was probably a combination of a desire to show membership of an elite, and partly that Greek is superior to Latin in its ability to express complicated thoughts precisely, to convey nuance, etc.[18]

---

summer of 480 BC. The Persian army ... (various figures are given by scholars ranging between about 100,000 and 150,000), arrived at the pass in late August or early September. The vastly outnumbered Greeks held off the Persians for seven days (including three of battle) before the rear-guard was annihilated in one of history's most famous last stands.

[16]Alexander was, with Napolean, one of the two great military leaders in history. He defeated Darius III at the Battle of Issus in 333 BC (S. Anatolia) and the Battle of Gaugamela in 331 BC (near Mosul, in modern Iraq). Alexander sacked Thebes early in his campaigns, and burned the Persian capital Persepolis later. Both Alexander and Napolean were keen students of mathematics, and of geometry in particular.

[17]The Coliseum in Rome is perhaps the best known, though it is partially ruined. By contrast, the Pont du Gard (a Roman aqueduct near Arles) is in excellent condition.

[18]Julius Caesar's two most famous sayings are "*Iacta alea est*" (the die is cast), when by crossing the Rubicon (from Gaul to Italy, with his army, 49 BC) he committed himself to challenge the political power of the Roman Senate, and his last words, "*Et tu, Brute*" (You too, Brutus, 44 BC). He may have said either or both in Greek. [Beard, 337-8: 'As he fell, Caesar cried out in Greek to Brutus 'You too, child', ... The famous Latin phrase *'Et tu, Brute'* ... is an invention of Shakespeare's.]