m3hprob1.tex

M3H HISTORY OF MATHEMATICS: PROBLEMS 1. 16.1.2018

Q1. Prove the theorem of Thales: an angle in a semicircle is a right angle. [You are put on your honour here *not* to consult any written source.]

Q2. Prove the theorem of Pythagoras: in a right-angled triangle, the square on the hypotenuse is the sum of the squares on the other two sides.
[Again: *no* written sources.]

Q3. Prove (Euclid Book I, Prop. 32) that the angle sum of a (plane) triangle is π. [Ditto.]

Q4 *Star pentagram and golden section* (Euclid Book 6, Prop. 30; cf. the Platonic solids).

In a regular pentagon $ABCDE$ of side a, join up each vertex to its two opposite vertices. The resulting figure is the *star pentagram*, and contains an inner pentagon $A'B'C'D'E'$ say (with A' the vertex opposite A, etc.), of side $a - b$ say (so $b = AB' = AE'$, etc.).
(i) Show (using similar triangles AED, $B'ED$, or otherwise) that

$$\frac{a+b}{a} = \frac{a}{b} =: \phi,$$

say, where the *golden ratio* ϕ is given by

$$\phi = \frac{1}{2}(1 + \sqrt{5}).$$

(ii) Show that the ratio of the sides of the two pentagons is

$$\frac{a-b}{a} = 1 - 1/\phi = \frac{1}{2}(3 - \sqrt{5}).$$

(iii) Show that

$$\phi = 2\cos(\pi/5) = 1 + 2\sin(\pi/10).$$

NHB