m3hprob4.tex

M3H PROBLEMS 4. 6.2.2018

Q1 *Calculation of* π, *after Tse Chung-chi (430-501 AD).*
(i) In a circle of radius r, let $PQ = s$ be a side of a regular inscribed n-gon. Let M be the mid-point of PQ, $u := OM$, OM produced meet the circle in R, $v := MR$, $w := RQ$. So w is the side-length of a regular inscribed $2n$-gon (draw a diagram).

By applying Pythagoras' theorem to triangles OMP and MRQ, or otherwise, show that

$$w^2 = 2rv.$$

(ii) By taking $r = 1$, show that the iteration

$$s \to u := \sqrt{1 - (\frac{1}{2}s)^2} \to w := \sqrt{2(1-u)}$$

takes the side of such an n-gon into that of such a $2n$-gon.
(iii) Hence obtain π to the limits of accuracy of your pocket calculator.

Q2 (*Fibonacci sequence*).

Find the nth Fibonacci number u_n. Show that

$$u_{n+1}/u_n \to \phi := \frac{1}{2}(1+\sqrt{5}),$$

the golden section.

Q3 (*Long division: Fibonacci (1170-1250), Liber Abaci, 1202*).
(i) If $x = m/n$ is a rational in its lowest terms, show that its decimal expansion terminates or recurs in at most $n - 1$ places.
(ii) Show that x is rational iff its decimal expansion terminates or recurs.
(iii) Find the decimal expansions of $1/7, 2/7, 3/7, 4/7, 5/7, 6/7$, and comment.

NHB