m3hw10.tex **Week 10. 15-20.3.18**

20th C.: Early

We move now to the 20th C. Week 10 will focus on the first half, Week 11 (not taught – for info) on the second half. The core of the undergraduate curriculum is based on the 19th C. The mathematics of the 20th C. is different: there is much more of it (in line with population growth, etc.); it is much more technical (so harder to even sketch at this level). Consequently, no single person can hope to have a proper understanding of *all* of 20th C. mathematics. So inevitably, our treatment is incomplete, and reflects my own knowledge and interests.

The 20th C. has seen the emergence of the modern world: from 1900 (electricity supply and the telegraph were both new) to the 'global village' of the world in the internet age.

In 1900, the UK had only a handful of universities, and most of them did not admit women. Universal adult suffrage, even in the West, is post-WWI. Only a tiny fraction of the school cohort went to university. Similarly in Germany and France, the leading countries mathematically at that time.

US mathematics took off between the World Wars; before then, it was normal for US academics to take their PhDs in Europe, usually Germany.

The great change in the mathematical world was the great change in the world generally: the Nazis, and later WWII. A high proportion of leading academics then as now were Jewish; they were forced out of their jobs in Germany by state decree, and either fled to the US, UK etc. or (largely) perished in the Holocaust. Intellectual leadership passed irrevocably from Europe to the USA, as a direct result of Adolf Hitler (1889-1945).

WWI led to the collapse of four empires (German, Austrian, Russian, Ottoman), and WWII to that of a fifth (British). The Russian Empire fell in 1917; the USSR, which replaced it, fell in 1991. The resulting turmoil was traumatic for the fSU – but had a wonderful effect on western mathematics, as many brilliant and superbly educated mathematicians were able to seek employment elsewhere, where they would be paid in hard currency. Hence the large number of Russian names in the staff lists of western universities.
Note. Jewish scientists have always been prominent out of all proportion to their numbers. Jews constitute 0.2% of the world's population, but 20% of Nobel prize-winners (presumably because of the Jewish respect for learning).

ANALYSIS

Emile Borel (1871-1956)

Borel took his PhD in 1893, supervised by Darboux.

Leçons sur les séries divergentes, 1895 (2nd ed. 1928).

Leçons sur la theorie des fonctions, 1898 (2nd ed. 1914, 3rd ed. 1928).

Leçons sur les fonctions monogènes uniformes d'une variable complexe, 1917.

In these and other books, in his lectures and in the work of his students, Borel seeks to systematise the treatment of analysis, post-Cantor. From 1905, he also worked on probability theory.

Henri Lebesgue (1875-1941)

Lebesgue took his PhD in 1902, supervised by Borel. In his pioneering thesis 'Intégrale, longueur, aire' (*Annali di Mat.* **7** (1902), 231-259), Lebesgue introduces the new subject of *measure theory*, and a new integral, now called the *Lebesgue measure*. Despite being harder to set up than the Riemann integral, it is much more general and powerful and much easier to manipulate (e.g., in interchanging limit and integral – *Lebesgue's monotone convergence theorem* and *Lebesgue's dominated convergence theorem*). Lebesgue measure is the mathematics of length, area and volume. It is also the mathematics of gravitational mass, electrostatic charge, and probability; in probability, the total mass is one, and the integral is the expectation. The essence of measure theory is *countable additivity*: for A_n disjoint measurable sets with measures $\mu(A_n)$, the measure of their union is the sum of their measures:

$$\mu(\bigcup_0^\infty A_n) = \sum_0^\infty \mu(A_n).$$

René Baire (1874-1932)

Baire's thesis (1899) contains the *Baire category theorem*. This is now stated in the language of General Topology, which did not then exist, so we will not go into detail. Suffice to say here that category and measure are closely linked. For a monograph comparing them, see

J. C. OXTOBY, *Measure and category*, Springer, 1971 (2nd ed. 1980).

Maurice Fréchet (1878-1973)

Fréchet's thesis of 1906, written under Hadamard, introduced *metric spaces* (not under that name). This is an important step towards General Topology (below). Fréchet's work marks a new level of *abstraction* in mathematics; this will be characteristic of the 20th C.

Felix Hausdorff (1868-1942).
Grundzuge der Mengenlehre, 1914 (2nd ed. 1927, 3rd ed. 1937)

In this book, Hausdorff begins the subject of General Topology (including as a special case metric spaces, which he named). This provides the right language in which to discuss 'open and 'closed' – crucial for Analysis, as we have seen (e.g. Week 9, Heine's theorem, Heine-Borel theorem).

General Topology was systematically studied by the new Polish school of mathematics[1], principally by Kazimierz Kuratowski (1896-1980). In honour of this, a complete separable (i.e., having a countable dense set, like the rationals) metric space – the right setting in which to do Analysis – is called a *Polish space.*

Constantin Carathéodory (1873-1950).
Theory of functions of a complex variable, I, II 1950/1960.;
Conformal representation, Cambridge Tracts 28, CUP, 1932 (2nd ed. 1958)

Carathéodory was the greatest Greek mathematician since antiquity, though he spent most of his career in Germany. He was basically an analyst, but had a wide range, and was regarded as the natural successor to Hilbert in German mathematics. The Carathéodory extension theorem is the key to technical Measure Theory. He also worked on thermodynamics, conformal representation, and calculus of variations.

FUNCTIONAL ANALYSIS (FA)

As we have seen (Week 9), *Hilbert space* emerged in 1905 in Schmidt's work, and Hilbert-Schmidt operators on them in 1908. This led to a new ability to expand solutions of ODEs, PDEs and integral equations in series of *eigenvalues* and *eigenfunctions.* This led to the subject of *spectral theory*, and the Courant-Hilbert book of 1924. This was just the mathematics needed to handle the demands of *quantum theory*, which began its modern form in 1925. The year 1932 saw three very different key books:
M. H. STONE, *Linear transformations in Hilbert space*, AMS Coll. Publ. 15 (1932);
J. von NEUMANN, *Mathematische Grundlagen der Quantenmechanik*, Springer

[1]Poland achieved independence after WWI, having been partitioned between Austria, Prussia and Russia.

(1932);
S. BANACH, *Théorie des opérations linéaires*, Warsaw (1932).

Stefan Banach (1882-1945)

The modern era of FA begins with Banach's book. Hilbert space is a generalisation of Euclidean space to infinitely many dimensions; in both one has an *inner product* (generalising the ordinary dot product of vectors), and the length (or *norm*), metric and topology stem from that, Banach recognised that this is too special, and studied complete normed vector spaces (topological vector spaces in which the topology is given by a norm and is complete). These spaces are very important, and are named *Banach spaces*. The *Hahn-Banach theorem* is one of the cornerstones of FA.

Laurent Schwartz (1915-2002)

Schwartz was an analyst, best known for his *generalised functions* (or Schwartz distributions) of 1950/51. These made rigorous the "Dirac delta" of quantum mechanics, used freely in Physics from 1930 on. This broadening of the definition of a function leads to a much cleaner theory of PDEs.

ANALYTIC NUMBER THEORY

We met the Prime Number Theorem (PNT in Week 8, with Gauss and Legendre, where it remained a conjecture. Just as 1872 saw two independent constructions of $\mathbb{R}$, so 1896 saw two independent proofs of PNT, due to the Frenchman Jacques Hadamard (1865-1962) and the Belgian Charles de la Vallée Poussin (1866-1962) (*Cours d'analyse* 1 (1902), 2 (1906), 7th ed. 1938; *Intégrales de Lebesgue, fonctions d'ensemble, classes de Baire*, 1916).

Edmund Landau (1877-1938), PhD 1899, Berlin (Frobenius and Fuchs)
Handbuch der Lehre von der Verteilung der Primzahlen (1909) [Handbook on the theory of the distribution of prime numbers]
Vorlesungen über Zahlentheorie (3 vol.) (1927) [Lectures on number theory].

Landau is credited (by Hardy) with having created Analytic Number Theory as a subject. For details, see e.g. NHB, M3P16.

Landau's extensive mathematical writings were admired for their style as well as their content. The *Satz-Beweis* (theorem-proof) style usual today – though traceable back to the ancient Greeks – takes its modern form as much from Landau as from anyone.

Landau's *Ergebnisse* (1916, 2nd ed. 1986) was an excellent and influential book on Complex Analysis.

HARDY and LITTLEWOOD
G. H. Hardy (1877-1947); *Works* I-VII;
J. E. Littlewood (1885-1977); *Works* I, II.

When Hardy was a student in Cambridge in the 1890s, Britain was not known for analysis. Hardy, an analyst and number theorist, became the best-known British mathematician of the 20th C. Littlewood too was an analyst. The Hardy-Littlewood collaboration (95 papers, from 1912 to 1948) is the longest and most famous in mathematical history. Both wrote excellent autobiographies:
G. H. HARDY, *A mathematician's apology*, CUP, 1940 (repr. 1979);
J. E. LITTLEWOOD, *A mathematician's miscellany*, 1953 (2nd ed., *Littlewood's Miscellany*, ed. B. Bollobás, 1986).

Srinivasan Ramanujan (1887-1920), FRS and Fellow of Trinity College, Cambridge, 1918.

Ramanujan was a self-taught genius and the greatest of Indian mathematicians. He was principally a number theorist. In 1913, as a clerk in Madras, he wrote to Hardy listing some of his best results. Hardy and Littlewood recognised his talents, and got him to Cambridge in 1914. He fell ill there in 1917, and returned to India in 1919, where he died the next year.

Ramanujan's work is too technical even to attempt to summarise here. Hardy's lectures on Ramanujan's work were published in 1936 (Inst. Adv. Study, Princeton). More recently, the mathematicians G. E. Andrews and B. C. Berndt have developed his work, edited his notebooks, etc.

Norbert Wiener (1894-1964); *The Fourier integral*, CUP, 1933

Wiener was a wonderful analyst and an all-round genius, who contributed to many areas. For appreciations of his work, see e.g.
Proc. Norbert Wiener Centenary Congress, 1994, PSAM 52, AMS, 1997;
The legacy of Norbert Wiener: A centennial Symposium, AMS 1997.
See also his autobiography, *Ex-prodigy* and *I am a mathematician*, MIT Press, 1953, 1956.

RUSSIAN ANALYSIS
N. N. Luzin (Lusin) (1883-1950)
Les ensembles analytiques, 1930 (with a foreword by Lebesgue).

Luzin was an analyst, and (with his student *M. Ya. Suslin* (Souslin) (1894-1919), who died tragically young of typhus) one of the creators of the theory of *analytic sets*. These arose out of a mistake in Lebesgue's work, and now play a key role in *descriptive set theory*. See e.g.
A. S. KECHRIS, *Classical descriptive set theory*, Springer, 1995.

Andrei Nikolaevich Kolmogorov (1903-1987)
Grundbegriffe der Wahrscheinlichkeitsrechnung, Springer, 1933 [Foundations of probability theory].

Kolmogorov was one of the greatest mathematicians of the 20th C., and the greatest probabilist ever. He was a pupil of Luzin, and began as an analyst. He became famous early for his work on Fourier series. His *Grundbegriffe* founded modern Probability Theory as a part of Measure Theory, where the (probability) measure has total mass 1. His deepest single result is probably his solution to Hilbert's 13th problem. For details of his extraordinary career, see e.g.
D. G. KENDALL, Obituary, A. N. Kolmogorov, *Bull London Math. Soc.* **22** (1990), 31-100.
This contains ten pieces on different aspects of his work.[2] For an account of the passage from Lebesgue to Kolmogorov here, see e.g.
NHB, Studies in the history of probability and statistics XLVI. Measure into probability: from Lebesgue to Kolmogorov. *Biometrika* **87** (2000), 145-156.

LOGIC AND FOUNDATIONS

Following Cantor's introduction of set theory in the late 19th C., it was soon noticed (by Burali-Forte and others) that one can be led into contradiction if one proceeds naively. An example is the 'paradox of the liar'. To see one form of this, write 'The statement on the other side of this piece of paper is false' on both sides of a piece of paper. Now read one side, and ask yourself whether what you read is true or false (it can be neither). For another form, the 'barber's paradox', see Kline, 1183.

This has become known as *Russell's paradox*, after the work of Bertrand Russell (1872-1970). Russell developed his 'theory of types' to avoid this. [See e.g. Halmos' *Naive set theory*.] Russell set out his theory in *Principles of Mathematics* (1903), and later (with A. N. Whitehead (1861-1947)) *Prin-*

[2]NHB, Kolmogorov's work on limit theorems in probability theory, 51-58; see also *Th. Probab. Appl.* **34** (1989), 129-139.

cipia Mathematics (3 volumes, 1910-13). See also Hilbert's second problem.

The axioms of 19th C. mathematics needed to be augmented for technical reasons. One way to do this is via the *Axiom of Choice (AC)*, introduced by Ernst Zermelo (1871-1953). It is more usually employed in an equivalent formulation, *Zorn's Lemma* (Max Zorn (1906-93)) (cf. Hilbert's first problem). AC is needed for the Hahn-Banach theorem, so in Functional Analysis, etc.

As noted (Week 9), Hilbert's views on the nature of mathematics and its foundations were too naive. Kurt Gödel (1906-78) showed in 1931 that mathematics is *incomplete*: any mathematical theory rich enough to contain the set $\mathbb{N}$ of natural numbers must contain statements which can be neither proved nor disproved. As a corollary, the consistency question raised by Hilbert is undecidable (Kline, 1206-7; B 27.10). Gödel also showed in 1940 that AC is *consistent* with the other axioms of set theory. Alfred Tarski (1902-83) was a Polish Jew who was in the US at the start of WWII, and stayed there; he was a logician, working on e.g. model theory. He and Banach introduced the *Banach-Tarski paradox* in 1924 (not really a paradox!).

Alan M. Turing (1913-54) showed in 1937 that Hilbert's question on decidability has a negative answer. His work led to the development of *computable numbers*, and later to the development of the *computer*. See e.g.
A. HODGES, *Alan Turing – the enigma of intelligence*. Counterpoint, 1983.

TOPOLOGY

We have discussed General Topology above under Analysis, and the work of Hausdorff and Kuratowski. For a full history of topology, see
I. M. JAMES (ed.), *A history of topology*, North-Holland, 1999 (40 ch.).
See also the topologists in James' *Remarkable mathematicians* (W 0).
L. E. J. Brouwer (1881-1966)

Brouwer was a Dutch topologist, who in 1909 proved his *fixed-point theorem*; see e.g. Th. 1.9 of A. HATCHER, *Algebraic topology*, CUP, 2002. Brouwer lost faith in his early work, and in mathematics in general, and devoted himself to *intuitionism*, a movement in Mathematical Logic (which, like Constructivism, sought to avoid non-constructive proofs, proof by contradictiion, etc.). This led to a struggle with Hilbert, who was editor-in-chief of the *Mathematische Annalen*; Hilbert got Brouwer expelled from the editorial board (the *Annalenstreit*).

A definitive early classic was
P. Alexandroff & H. Hopf, *Topologie*, Springer, 1935.

ALGEBRA

Our general references are

G. BIRKHOFF & S. Mac LANE, *A survey of modern algebra*, Macmillan, 1953;

P. M. COHN, *Algebra*, Vols 1-3, Wiley (1, 1974/82; 2, 1977/89; 3, 1991).

Group representations (Frobenius, Burnside, 19th C.) were taken further by *Issai Schur* (1875-1941). They are widely used in quantum mechanics:

B. L. van der WAERDEN, *Die gruppentheoretische Methode in der Quantenmechanik*, Springer, 1932.

Emmy Noether (1882-1935)

Emmy Noether, a German Jewess, was arguably the greatest woman mathematician of all time; her work was of prime importance in the development of modern algebra. She eventually obtained an academic post at Göttingen, with support from Hilbert, in the teeth of opposition to the appointment of a woman. Then, being Jewish, she had to flee Germany, and worked in the US until her untimely death.

B. L. van der Waerden (1903-96)

Moderne Algebra, I (1937), II (1940), Springer.

Van der Waerden was a Dutch mathematician who spent most of his career in Germany. He was much influenced by Noether, and much of her work is incorporated in his Volume II.[3]

We return to the links between Algebra and Topology in Week 11; cf.

Ch. A. WEIBEL, *History of homological algebra*, his webpage, Rutgers U.

GEOMETRY

The 19th C. work on geometry was continued by the Italian school. In particular, Ricci and Levi-Civita revolutionised differential geometry by introducing tensor methods in the early 20th C. This provided the mathematial machinery needed for Einstein's General Theory of Relativity (below).

H. F. Baker (1866-1956) of Cambridge continued and developed these ideas; see e.g. his obit. [JLMS 32 (1957), 112-128, Hodge]. He was succeeded in 1936 by W. V. D. (Sir William) Hodge (1903-75) [obit. BLMS 9 (1977), 99-118, Atiyah]. The non-technical parts of this article provide a good account of the increasingly close connections between geometry and topology; the two subjects are now all but inseparable. Note the interpretation of Maxwell's equations in terms of Hodge's theory of harmonic forms.

[3] *Moderne Algebra* uses Fraktur, or 'Gothic letters' ('backslash mathfrak' in TeX). See W for this, and a history of the 'Antiqua-Fraktur' controversy under the Nazis.

QUANTUM THEORY

Black-body radiation. Classical physics in the 19th C. had many successes, including the discovery of the *electron* in 1897 by J. J. Thomson (1856-1940). However, it failed to explain some phenomena, such as the observed variation of energy with frequency in black-body radiation. *Max Planck* (1858-1947) inaugurated modern physics in 1900 by proposing a *quantum theory* of radiation (in which energies take discrete rather than continuous values). Using thermodynamic arguments, he introduced *Planck's constant* $\hbar$, and measured it experimentally. His work led Kuhn towards his theory of *paradigm shifts*: Th. S. KUHN, *Black-body theory and the quantum discontinuity*, 1894-1912. U. Chicago Press, 1978.

Photoelectric effect. Hertz (1886/7), while experimentally confirming Maxwell's electromagnetic theory of light, discovered the *photoelectric effect*: ultraviolet light causes electrons to be omitted from a metallic surface. Experiment shows that there is a minimum (threshold) frequency below which emission ceases. Classical physics cannot explain this, but Einstein (below) proposed a *quantum* explanation (radiation is itself discrete: the packets are called *photons*) in 1905. This led him to a value of $\hbar$ in agreement with Planck's.

Atomic spectra. It was known classically that atoms emit electro-magnetic radiation (e.g. when heated), and that the frequencies are discrete and characteristic of the type of matter. Following the nuclear model of the atom proposed by Ernest Rutherford (1871-1937) in 1911, Niels Bohr (1885-1962) suggested that electrons have stationary orbits around the nucleus, their angular momenta being quantised (i.e. discrete). This was explained in 1924 by Louis de Broglie (1892-1987) in terms of *wave-particle duality*: both radiation and matter can be considered as either waves or particles.

Modern quantum theory began in 1925 with the work of Werner Heisenberg (1901-1976) and Erwin Schrödinger (1887-1961). Their approaches were formulated differently, Heisenberg's in terms of matrices, [4] Schrödinger's in terms of PDEs/waves, but were shown to be equivalent by Schrödinger in 1926. Heisenberg's main contribution is his *Uncertainty Principle*: it is impossible to measure both the position and the momentum of a particle exactly. Schrödinger's main contribution is his PDE, *Schrödinger's equation*.

When written as an eigenvalue problem, the eigenvalues give the allowed energies of the system. *Pauli's exclusion principle* (Wolfgang Pauli (1900-58)

[4] When Heisenberg found his matrix formulation, he didn't know what a matrix was.

also dates from 1925, as do the *Pauli spin matrices* (cf. quaternions).

Paul Dirac (1902-84); *Principles of quantum mechanics* (1930), Nobel Prize 1933 (with Schrödinger): Dirac gave a relativistic treatment of the electron, and predicted the *positron* and *anti-matter*.

Group theory in quantum mechanics

The representation theory of topological groups (particularly the classical compact groups) play a key role in quantum mechanics (via the Peter-Weyl theorem). This led to group theory invading physics, and to physicists of the time talking about the *Gruppenpest* (group plague). The first book on this was by Hermann Weyl (1885-1955):

H. WEYL, *Theory of groups and quantum mechanics*, (German, 1928, 1930); Eng. tr. 1931.

H. WEYL, *The classical groups: Their invariants and representations*, 1939/53; *symplectic group* (arising from Hamiltonian mechanics); *Lie algebras*.

Spin entered Quantum Mechanics in 1927 with Pauli; Dirac used it in 1928. *Spinors* were studied by Elie Cartan (1869-1951) (*The theory of spinors*, 1937/66/81, Dover). He also worked on classification of Lie algebras, and introduced the *exterior derivative* in differential geometry.

RELATIVITY

Tullio Levi-Civita (1873-1941), pupil of Ricci.

Riemann's theory of manifolds was developed by Christoffel, Bianchi and others. Ricci and Levi-Civita (Math. Ann. 54 (1901), 125-201) studied differential invariants on manifolds: invariant under changes of coordinate system, and so properties of the manifold (or the physical system it represents). The name *tensor calculus* is from Einstein in 1916.

Albert Einstein (1879-1955)

Einstein worked in the Swiss Patent Office in Berne from 1902. He became Associate Professor of Theoretical Physics at ETH, Zürich in 1909, and Professor in Berlin in 1914. He won the Nobel Prize for Physics in 1921 (for work on the photoelectric effect – relativity theory was considered too controversial still). A Jew, he left Germany in 1933 for the new Institute for Advanced Study in Princeton, New Jersey.

Einstein was born in Ulm, which celebrated *Einstein Year* in 2005, for his three great papers of 1905: on the photoelectric effect, the molecular explanation of Brownian motion, and the Special Theory of Relativity. This introduces the Einstein equation $E = mc^2$, and unifies Newtonian mechanics and Maxwellian electrodynamics for bodies in uniform (relative) motion).

The *General Theory of Relativity (GTR)* (announced 1915, publ. 1916) extends the Special Theory to cover acceleration, e.g. under *gravity*.

The GTR makes three key testable predictions:

1. *Perihelion of Mercury* (perihelion: point of closest approach to the Sun). This was known to advance by about 574 seconds of arc per century ('relative to the fixed stars'). There was a discrepancy of 45 ± 5 seconds of arc between classical (Newtonian) theory and observation. GTR successfully explained 43 seconds of arc (1915).
2. *Bending of light by a gravitational field.* GTR's prediction was confirmed experimentally by A. S. (Sir Arthur) Eddington (1882-1944) during an eclipse of the Sun on 29 May 1919. This made Einstein world-famous overnight, and helped restore scientific relations with Germany after WWI. [5]
3. *Gravitational shift of spectral lines.* See Whittaker, EM II, 180-181.

Einstein was the greatest theoretical physicist since Newton, and was perhaps the last mathematician (except Hawking, Week 11) to be famous – in the ordinary sense, of being well known to the general public.

FLUID MECHANICS

The theory of hydrodynamics goes back to Daniel Bernoulli and Euler; the Navier-Stokes equations of viscous flow were known by 1850. The advent of the steam engine gave a new impetus, to naval architecture, and to the flow of pumped water in pipes. The 20th C. has been much concerned with aeronautics. We return to this in Week 11.

STATISTICAL MECHANICS

From Clausius, entropy increases; from Kelvin, equilibrium minimises energy. These tendencies are often in conflict with one another; hence *phase transitions* – freezing and boiling, ferro-magnetism, etc., and interface phenomena such as *surface tension.* Following the *Ising model* (Ernst Ising (1900-98) in 1925) for ferromagnetism, Lars Onsager (1903-76) solved the 2-dimensional Ising model with no magnetic field in 1944. An important tool here is the *Legendre transform* (see Week 9: Georgii and Ellis).

Quantum theory raised new problems. Particles may be *bosons* (Bose-Einstein statistics: symmetric wave-function, integer spin; do not satisfy Pauli exclusion; prototype: photon) or *fermions* (Fermi-Dirac statistics: anti-symmetric wave-function, half-integer spin; satisfying Pauli exclusion; prototype: electron). For background, see e.g.

B. SIMON, *The statistical mechanics of lattice gases, Vol. I*, PUP, 1993.

[5] Hardy was British, Marcel Riesz Hungarian. Their 1915 book on Dirichlet series bears the lovely inscription 'Auctores hostes idemque amici' – Authors, enemies and yet friends.

STATISTICS

After the discovery of the method of least squares, and the Gauss-Laplace synthesis (the First Heroic Period of 19th C. Statistics), there was a Long Pause, in which much else was done in mathematics but little in statistics. The Second Heroic Period was inaugurated by Sir Francis Galton (1822-1911), with his study of *Hereditary genius* (1869). This led to the concepts of *regression* and *correlation*. See e.g. NHB, *Heroic Periods* (Papers), NHB and J. M. Fry, *Regression*, Springer, 2010.

The relevant mathematics was developed by Karl Pearson (1857-1936), who inaugurated the modern era in Statistics with his *Chi-square* (χ^2) test of goodness of fit in 1900, and F. Y. Edgeworth (1845-1926), who gave the multivariate normal density in 1893. This paved the way for the career of R. A. (Sir Ronald) Fisher (1890-1962), the greatest statistician ever, *and* one of the greatest geneticists. Fisher did his greatest work at the Rothamsted Experimental Station in Harpenden, Herts in the 1920s. From the Dramatis Personae in Bingham & Fry:

R. A. (Sir Ronald) Fisher (1890-1962), likelihood, 1912 [1.6], density of r^2, 1915 [7.3], F-distribution, 1918 [2.6], ANOVA, 1918 [Ch. 2], sufficiency, 1920 [4.4.1], z-transformation, 1921 [7.3], maximum likelihood, 1922 [1.6], Fisher's Lemma, 1925 [2.5], ANCOVA, 1932 [5.2], ancillarity, 1934 [5.2], information matrix, 1934 [3.3], design of experiments, 1935 [9.3], scoring, 1946 [8.1]

GENETICS

Mathematics and Biology.

The two foundations of modern biology are the Darwinian theory of natural selection (Charles DARWIN (1809-1882): *On the Origin of Species by means of Natural Selection*, 1859 – *The Origin of Species*), and Mendelian genetics (Gregor MENDEL (1822-1884): *Experiments on plant hybridization*, 1866). Mendel's work was largely forgotten, but rediscovered in 1900. It was thought at first that Mendelian genetics and Darwinian natural selection were incompatible, but this is not so; the two were synthesized by three people: the American Sewall Wright (1889-1988) and the Englishmen Fisher (above) and J. B. S. Haldane (1892-1964). The resulting Darwin-Mendel synthesis is the basis of modern biology.

The relevant mathematics used *Markov chains* (*A. A. Markov* (1856-1922) in 1907). These give models for stochastic processes (random phenomena unfolding with time) in which there is no memory. The *Fisher-Wright model* of mathematical genetics is one of the most important classical Markov chains.