m3hw4.tex **Week 4. 1-6.2.18**

THE GREEKS (POSTSCRIPT); THE ROMANS; THE ARABS; INDIA and CHINA

Farewell to Pappus, and to Alexandria

The Golden Age of Greek geometry had ended with Apollonius some 500 years before Pappus. Of Pappus, Heath writes (p. 355) '... the great task he set himself was the re-establishment of geometry, on its former high plane of achievement. Presumably such interest as he was able to arouse soon flickered out, but for us his work has an inestimable value as constituting, after the works of the great mathematicians which have actually survived, the most important of all our sources.'

Pappus was the last of the major Greek geometers, ending a tradition spanning some eight centuries. He was also the last great Alexandrian mathematician (though we should also remember the martyrdom of Hypatia, 415 AD: B, p.213). No other centre of learning has ever dominated mathematics for as long (the six centuries from Euclid to Pappus) as did Alexandria.

History of Mathematics: Early sources

The first account of the history of mathematics of which we know was written by *Eudemus* (of Rhodes, fl. c. 320 BC: Week 3), a student of Aristotle. Though lost, this was (in part) summarised (e.g. by Proclus, below). It is from Eudemus, through Proclus, that we have much of our knowledge of early Greek mathematical history (e.g. pre-Euclidean geometry).

Proclus (of Alexandria, 410-485 AD).

Proclus taught at Athens as head of the 'Neoplatonic school', and wrote:
(i) the *Eudemian sumary*, his account of Eudemus' work above;
(ii) his *Commentary* on Euclid; see Heath II, Ch. XXI.

The Abacus

J. M. PULLAN, *The history of the Abacus*, Hutchinson, London, 1968
Parry MOON, *The Abacus*, Gordon and Breach, London, 1971

Herodotus (of Halicarnassus, Asia Minor, c. 484 – c. 425 BC), the Greek historian regarded as the 'father of history', records the use an abacus to solve arithmetical problems, and gives one such: find the interest due after 1,464 days on a principal of 766 talents, 1,095 drachmas and 5 obols, at a rate of 1 drachma per day per 5 talents (1 t. = 6,000 d.; 1 d. = 6 o.).

Early counting was done by means of small stones or pebbles (calculus = pebble, Latin; hence calcaria = limestone; chalk; calcium). Calculations were done by arranging pebbles on a flat surface (abax = flat surface, Greek;

hence abacus). Boards divided up into squares, as with chess or draughts, were often used for such purposes. The chess board (écheque in French) gave us the name Exchequer, from Norman French.

The Greeks excelled in trade, and were adept at the calculations needed for commerce. By contact with Greek culture, before and after the conquest of Greece by Rome, Greek use of calculating techniques and devices became available to the Romans, who absorbed them.

The term 'abacus' today refers to a frame containing parallel wires on which beads can move. Such abaci were in use in my lifetime (and at the time of writing of the two books cited above) in China, Japan etc., though the widespread availability of pocket calculators from the mid-70s on (made possible by the development of semi-conductors) has made them obsolete.

The upshot of all this is that calculating devices needed for the purposes of trade and administration were widely available, and widely use, throughout the advanced civilisations of the ancient world.

Overall assessment of Greek Mathematics

G. H. HARDY, *A mathematician?s apology*, CUP, 1940, §8, p.20-21:

S. BOCHNER, *The role of mathematics in the rise of science*, PUP, 1966.

Thucydides (c. 460 – c. 395 BC), *History of the Peloponnesian War*.

We have repeatedly stressed our debt to the Greeks. We owe to them the concept of Mathematics as a subject, rather than a bag of computational techniques; the idea of proof; the pursuit of knowledge for its own sake rather than for utilitarian purposes; much knowledge of astronomy, including the heliocentric view; much geometry, some algebra, number theory, and integral calculus (not by that name), etc.

Hardy [above; W10] writes: 'The Greeks were the first mathematicians who are still 'real' to us today. Oriental mathematics may be an interesting curiosity, but Greek mathematics is the real thing. The Greeks first spoke a language which modern mathematicians can understand; as Littlewood [also W10] said to me once, they are not clever schoolboys or 'scholarship candidates' but 'Fellows of another college'. So Greek mathematics is 'permanent', more permanent even than Greek literature. Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. 'Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean.'

We will turn shortly to the most obvious reason for the end of Greek dominance in mathematics – the rise of Rome.

We turn now to an assessment of the limitations of Greek mathematics.

Example: The Fundamental Theorem of Arithmetic (FTA)

We learn in primary school that any natural number can be expressed as a product of prime factors, essentially uniquely. Then we learn about the hcf (highest common factor), lcm (lowest common multiple), etc. At university in the Mathematics curriculum, we return to foundational questions, and to FTA, armed with modern standards of rigour. See e.g. my homepage, link to M3P16 Analytic Number Theory, Lecture 1. From this:

"*Historical Note.* We owe Mathematics as a subject to the ancient Greeks. Of the 13 books of Euclid's Elements (EUCLID of Alexandria, c. 300 BC), three (Books VI, IX and X) are on Number Theory. From the ordering of the material in Euclid, it is clear that the Greeks knew that they did not have a proper theory of irrationals (i.e. reals). Although they did not state FTA, it had been assumed that they "knew it really", but did not state it explicitly. This view is contradicted by Salomon BOCHNER (1899-1982) (*Collected Papers*, Vol. 4, AMS, 1992). According to Bochner, the Greeks did *not* know FTA, nor have a notational system adequate even to state it!

L. E. DICKSON (1874-1954) (*History of the Theory of Numbers* Vols 1-3, 1919-23) does not address the question of the Greeks and FTA!

The first clear statement and proof of FTA is in Gauss' thesis ([W8]: C. F. GAUSS (1777-1855); *Disquisitiones Arithmeticae*, 1798, publ. 1801)." Bochner (p.214-6) credits this insight to conversation with G. H. Hardy in 1933; see Hardy and Wright, *Theory of Numbers*, 181 and 188.

Example: Zeno and motion

We mentioned Zeno and his 'paradoxes' [W2], in particular, Achilles and the tortoise. Zeno also had a 'paradox', concerning an arrow in flight, according to which the motion of the arrow was only apparent, not real. This is commented on by Aristotle (*Physics* VI.9, 239b5).

Zeno's arrow may serve as an illustration of a great deficiency in Greek mathematics, concerning *rates of change* in general and *motion* in particular. From my website, link to PfS (Probability for Statistics), Lecture 1 (immediately after the calculation $A = \pi r^2$ for the area of a circle):

Note. The ancient Greeks essentially knew integral calculus – they could do this, and harder similar calculations [volume and surface area of a sphere].

What the ancient Greeks did not have is *differential* calculus [which we all learned first!] Had they had this, they would have had the idea of velocity, and differentiating again, acceleration. With this, they might well have got Newton's Law of Motion, Force = mass × acceleration. This triggered the Scientific Revolution. Had this happened in antiquity, the world would have been spared the Dark Ages and world history completely different!

Greek mathematics had various structural defects which, at least, made it vulnerable to the fate which overtook it. We list a number.

1. *Notation*

Greek mathematical notation was inadequate in various ways: numerals, algebraic manipulation etc. See Bochner on FTA above (though Bochner also stresses the conceptual side here – it was not just a matter of notation).

2. *Practicality*

The mundane side of mathematics – e.g. practical calculation in everyday life, of money for example – lacked prestige in the eyes of Greek mathematicians. Euclid omits logistic, for example.

3. *Chronology*

The Greeks did not even evolve a convenient chronology. So they failed to introduce a coordinate system on the one-dimensional time axis – so even more, in two or three dimensions! This is one of the most glaring defects in the whole Greek heritage. For example, the historian Thucydides (whose *History of the Peloponnesian War* is superb), wanting to fix the date of an event, has (Bochner, p.53) 'to verbalize a monstrosity like this:
"When Chrysis had been priestess at Argos for 47 years, Aenesius being then ephor at Sparta, Pythodorus having yet four months left of his archonship at Athens" (Thucydides, ii, 2, I) = April, 431 BC.'
Even the Romans did better than this! They used dates AVC (from the foundation of the city of Rome – Ab Urbe Condita [separate letters for U and V came later]) until the present chronology AD (Anno Domini – in the year of Our Lord), after Emperor Constantine's conversion to Christianity.

4. *Sophistication*

The very sophistication of Greek mathematics rendered its writings hard to assimilate without human help (teaching). This teaching needed to be carried out in suitable institutions of higher learning – the Academy of Athens, the Museum of Alexandria, etc. These had to be supported, financially and politically, by society. So its continuing vitality depended on that of the surrounding society, in decline for geopolitical and economic reasons, military conquests etc.

The ordinary educated person is most aware of the ancient Greeks through their philosophers – e.g. Plato and Aristotle. It is arguable that many of the basic weaknesses of Greek mathematics are conceptual, and hence in part at least philosophical. This is basically Bochner's view (recall his description of Euclid as 'an insufferable pedant and martinet – Bochner, p.35).

ROME

H. H. SCULLARD, *A history of Rome down to the reign of Constantine*, Macmillan, 1975
Tom HOLLAND, *Rubicon: The triumph and tragedy of the Roman Republic*, Abacus, 2004 (Little, Brown 2003)
Mary BEARD, *SPQR: A history of ancient Rome*, Profile Books, 2015.

Rome gave nothing to mathematics, and indeed – by its negative effect on Greek culture – ended its Golden Age in antiquity and set civilisation back mathematically by a millennium. Roman numerals are so awful we shall not even discuss them. We discuss Rome nevertheless, as otherwise one cannot understand the flow of mathematics across cultures and ages.

Rome was traditionally founded in 753 BC, which (as noted above) gave the Romans an origin of time, dates being reckoned AVC, from then.

The Romans regarded themselves as being descended from Aeneas, who fled from the sack of Troy in Asia Minor. The Latin language (named after Latium, the region surrounding Rome) is Indo-European, and has links with the Hittite language (of ancient Asia Minor). Rome had kings until 509 BC, when Tarquin was expelled and a Republic set up. The Roman Senate (senex = old man in Latin – cf. 'senile') was the governing body in the Republic. Roman armies carried a standard with an eagle, the legion's number, and the inscription SPQR (Senatus populusque Romanus: the Senate and the people of Rome). The Romans learned much from other peoples (whom they conquered)– first the Etruscans, who lived in Tuscany, and later the Greeks. Horace: *Graeca capta ferum victorem cepit et artes intulit agresti Latio* ('Greece, once conquered, conquered her savage victor and brought culture into the rough land of Latium' – Beard, p.499).

Usually power was shared between two consuls, who served for a fixed term. But occasionally, in time of national emergency (sack of Rome by the Gauls, 390 BC; catastrophic defeat by Hannibal at the Battle of Cannae, 216 BC, Second Punic War), a dictator was appointed.

The political cohesion of the Republic frayed during the Civil Wars (88 – 80 BC) between Marius and Sulla (c. 138 - 79 BC), won by Sulla, who had himself declared dictator after – illegally – occupying Rome with his army. But Sulla then retired, shortly before his death.

Further political unrest followed. The Roman Republic was fatally undermined when Julius Caesar crossed the Rubicon and marched on Rome, 49 BC. He defeated Pompey in the resulting civil war, becoming dictator before his assassination in 44 BC. After further civil war, Caesar's adopted son Octavius became the first Roman Emperor, Augustus (reigned 27 BC - 14 AD).[1]

The Roman Empire

This began under the Republic. Roman influence in Italy expanded, culminating in the Pyrrhic War (280-275 BC), against Pyrrhus (319/8-272 BC), who had come to the aid of Tarentum (heel of Italy) against Rome. Pyrrhus won a costly victory at the Battle of Asculum in 279 BC (hence 'Pyrrhic victory'). Pyrrhus eventually withdrew from Italy, and became king of Epirus (across the Adriatic from the heel of Italy) and Macedon. After his death in battle, Greece and Macedon were vulnerable to Roman expansion.

Rome fought four wars against Macedon (214-205, 200-197, 171-168 and 150-148), after which Macedonia became a Roman province in 146. Greece came under Roman rule after the Battle of Corinth, also 146.

Rome inherited Pergamon in Asia Minor in 133 BC. In the Middle East, where there was a power vacuum as Seleucid rule decayed, Judea had fought wars against Greek-speaking neighbour city-states, in alliance with Rome. This led in 63 BC to negotiations between the Roman general Pompey and Antipater, under which Judea became a Roman client state.

Rome annexed Egypt in 30 BC after Octavius (later Augustus) defeated his former ally Mark Antony and Queen Cleopatra of the Ptolomaic dynasty at the Battle of Actium. Egypt remained a Roman province until 390 AD (a little before the time of Proclus, above).

Thus the Greek-speaking world, which had been the centre of mathematics for centuries, came under Roman rule. The mathematical traditions of the Greek world also came to an end. The connection seems clear enough.

The later history of the Roman Empire need not concern us here, partly because it is more familiar to us through our own history, partly because nothing much was happening mathematically.

Lucretius Carus (c. 99 – c. 55 BC); *De rerum natura* [On the nature of things]. Lucretius' poem contains the first documented reference to (what became known in the 19th C. as) *Brownian motion*. Lucretius observed dust particles dancing in sunbeams.[2]

---

[1] Hence the months July and August. This makes visible nonsense of the names of the remaining months, September ("the seventh month", now the ninth), October ("8th, now 10th"), November ("9th, now 11th") and December ("10th, now 12th").

[2] I remember seeing this myself, aged c. 4, and asking my mother why they did this.

INDIA

Background

Early civilisation in India was highly developed in Egyptian times, as is shown by the archaeological excavations at Mohenjo Daro in NW India. There may be links with Sumerian civilisation (the Hindu caste system may have Sumerian roots), as may the Aryan culture which produced Sanskrit, the language from which the Indo-European languages developed.[3]

The Sulvasutras ('rules of the cord') (B 12.11).

The 'rope-stretchers' of the Egyptian pyramid period have their counterparts in India. The Sulvasutra of Apastamba is a mathematical work dating back perhaps to Pythagoras' time. It contains Pythagorean triples, e.g. (5,12,13), (8,15,17), (12,35,37). These are thought to be of Mesopotamian origin.

The Siddhanthas, c. 400 AD (B 12.12)

The conquest (temporary and partial) of India by Alexander the Great exposed Indian science to Greek influence. It is known (e.g. from terminology: Indian names for planets, signs of the zodiac, etc.) that Indian astronomy had Greek origins. The earliest texts, the Siddhanthas, are works on astronomy in Sanskrit verse, with 'little explanation and no proof'. They contain an embryonic form of the sine function: emphasis is on, not angles and chords as in the Greek, but half-angles and half-chords.

Aryabhata, author of Aryabhatiya (499 AD) (B 12.13)

This is a handbook for measurement and astronomical calculation, containing: approximations to π; arithmetic progressions; areas and volumes, etc. (many of the results are incorrect). It also contains (B 12.14) work on measurement of time and spherical trigonometry. Here we find decimal-place notation: 'from place to place each is ten times the preceding'. The modern decimal numerals 1,2,3,4,5,6,7,8,9 are loosely called Arabic in English, but are called Hindu in Arabic; perhaps 'Hindu-Arabic' would be better. These evolved gradually; the key recognition that by use of place notation the same symbol could be used for three as for thirty, etc., had taken place by 595 AD (Indian source: date 346 in decimal notation), and in Western sources by 662 (Sebokt of Syria).

Please now ask yourselves this question.

[3]The 'family tree' of Indo-European or Sanskrit-based languages is interesting – see e.g. W. It includes all languages spoken in Europe except Basque, Hungarian and Estonian, and Finnish.

The zero symbol 0 came later (B 12.15). It had emerged by 876 in India (on an inscription in Gwalior), with the modern 0 for zero. The key components of (i) decimal base, (ii) positional notation, (iii) symbols for 0,1,2,...,9 were thus all in place. It seems that the Hindus did not invent any of them, but they did integrate them into (essentially) their modern form.
Trigonometry (B 12.16): The Siddhanthas and Aryabhatiya contain versions of what are essentially tables of sines.
Arithmetic (B 12.17-18). Hindu mathematics also contains versions of long multiplication and long division.
Brahmagupta (fl. 628) (B 12.19-20).

'Brahmagupta's formula' for the area of a cyclic quadrilateral is an extension of 'Heron's formula':

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

(sides a, b, c, d, semi-perimeter s). In fact, Brahmagupta did not give the restriction to the cyclic case (the correct general formula is

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta}$$

with θ half the sum of opposite angles). Note that the formula above is obviously wrong for a general quadrilateral (just take two opposite angles small).
Quadratics.

Brahmagupta's work is the first in which positive, negative and zero numbers are properly integrated into arithmetic. Exploiting this, he gave the general solution of the quadratic (apart from the complication of complex roots, which we take chronologically). *Diophantine equations* (B 12.21): General solution of the linear case $ax + by = c$.
Combinatorics.

'Pascal's triangle' was known in Indian as well as Chinese mathematics – centuries before Europe.
Power series

Series equivalent to the Maclaurin expansions of sin, cos and arctan were known in India more than two centuries earlier than their (re-)discovery in Europe. For details, see e.g.

D. Gold & D. Pingree, Historia Sci. 42 (1991), 49-65 (MR 92h:01009, R. C. Gupta);
C. T. Rajagopal & A. Venkataraman, MR 11-572;
R. C. Gupta, MR 58#26,730.

CHINA

Boyer Ch. 12; J. Needham, *Science and civilisation in China*, CUP
[Science and Civilisation in China (19542008) is a series of books initiated and edited by British biochemist and sinologist Joseph Needham (1900 - 1995). They deal with the history of science and technology in China. To date there have been seven volumes in twenty-seven books.]
Li YAN & Dù SHÍRÀN, *Chinese mathematics: A concise history* (tr. John N. Crossley & Anthony W.-C. Lun), OUP, 1987 ["Y&S" below]

Background

Chinese civilisation is almost five millennia old (c. 2,750 BC), but much of the early work is difficult to date accurately. This is complicated further by the mass burning of books ordered in 213 BC by the Emperor.

Early texts

I Ching (7th C. BC): 'Ying and yang': based on folklore and embodying superstition, but with some mathematics (permutations and combinations, etc.).

Nine chapters on the mathematical art (author and date unknown): a summary of mathematics from the Zhou and Qin to the Han dynasties (c. 11th C. BC – 220 AD) (Y& S 2.2). Like the Egyptian and Babylonian works, this is a collection of specific problems: no logical structure or proof (in contrast to the Greeks); no distinction between exact or approximate, etc.

Rod numerals (Y&S 1.2): decimal scheme, not sexagesimal as in Babylon; positional notation (several centuries BC). A blank space was used for zero. The calculations were set out much as with a Chinese abacus.

Calculation of π:

Liu Hui, 3rd C. AD: $\pi \sim 3.14$ (regular 96-gon); $\pi \sim 3.14159$ (3,072-gon)
Tse Chung-chih (430-501 AD): $\pi \sim 22/7$; $3.1415926 < \pi < 3.1415927$.

Precious Mirror (of the Four Elements), 1303 AD, *Chu Shih-Chieh*

Approximate calculation of roots of polynomial equations. Summation of series ($\sum_1^n r^2$, etc.). 'Pascal's triangle' of binomial coefficients.

Note. The name Pascal's triangle here is wildly anachronistic – Chu does not claim credit for it, referring to it as 'the old method'. It is known as Pascal's triangle because the West learned it through Pascal's book (see later) (Stigler's Law again).

THE ARABS

Albert HOURANI, *A history of the Arab peoples*, Faber, 1991.
Sir John GLUBB, *A short history of the Arab peoples*, Stein & Day, 1969 (General Sir John Glubb, 'Glubb Pasha' (1897-1986), author of 21 books).
B. L. van der WAERDEN, *A history of algebra – from al-Khwarizmi to Emmy Noether*, Springer, 1985.
Boyer Ch. 13; Dreyer Ch. XI

Background

The Roman Empire fell into decline, for many reasons. There were no natural boundaries, so emperors were tempted to over-expand and so over-stretch their resources. This led to growing military pressure from 'barbarians' on the fringes. There was recurring political violence, plague, economic problems etc. Emperor Constantine declared official toleration for Christianity in 313 AD; this later became the state religion of the Empire. He founded Constantinople (Byzantium) as capital of the Roman Empire (in the East) in 330 AD. Constantinople survived until 1453 (when it fell to the Ottoman Empire), but Rome fell in 410, when it was sacked by Alaric (370-410), King of the Visigoths (395-410). Wars were fought in an attempt to re-establish the Roman Empire in the West, led principally by the great general Belisarius (500 - 565) under Justinian, but from now on 'Emperor' meant the Emperor in the East, in Constantinople. In the West, the Roman Empire had been replaced by the Dark Ages.

Christianisation of the Empire went on in 4th – 6th C AD. Eventually the Catholic Church was established in Rome, with the Bishop of Rome (Pope) at its head, and the (Greek) Orthodox Church in the East. Doctrinal tensions emerged between the two, leading to endless conflicts (in the Crusades, the Crimean War, recently in Bosnia, etc.).

Early Christianity did not look kindly on mathematics and science. Hypatia, the first woman mathematician of whom we know, was murdered in Alexandria in 415 by a Christian mob (who mistook her learning for witchcraft). Emperor Justinian I closed the Academy of Athens in 529 AD.

In the East, the Byzantine Empire based on Constantinople ruled (or misruled) much of the Balkans, Asia Minor, the Middle East and North Africa. Much fighting took place between the Byzantine and Persian (Sasanid) Empires. By 600, both were exhausted, leaving a power vacuum in the East.

The superstition and ritual of prehistoric religion had been replaced by *ethical monotheism*, first in Judaism (which has roots some five millennia old, and its holy book, the Old Testament of the Bible), and later by Christianity, which developed from it, and has as its holy book the Bible (both Testaments, but it is the New Testament that is specifically Christian).

The Prophet Muhammad and Islam

The Prophet Muhammad (570-632)[4] introduced the third great religion of ethical monotheism, *Islam* (Arabic: resignation (to the will of God), hence Muslim), with its holy book, the *Qur'an* (Recitation).[5]

Islam achieved political power during the Prophet's lifetime. The powerful message of Islam, coupled with the political and religious decline in the E. Mediterranean and N. Africa, enabled the new religion of Islam, and the language of Arabic, to be spread by military conquest with great speed through the Middle East and N. Africa and into Spain. [When Alexander the Great conquered Egypt, he was welcomed as a liberator from the Persians. When in its turn the Byzantine Empire outstayed its welcome in Egypt, the population found their new masters less alien than their old ones.]

Decline and revival of learning

In the disruption of conquest, much of Alexandrian cultural achievement was lost. When Alexandria fell in 641, the Library of the Museum, the greatest in the world, was burnt – deliberately, according to legend.

As it expanded, the Arab world, too, began to fragment, evolving into the Western sphere, based on Spain, and the Eastern, based on Baghdad.

By 750, the caliphs (religious and political leaders) of Baghdad had begun to value learning (as happened later in W. Europe under Charlemagne, who became the first Holy Roman Emperor in 800). The caliph al-Mamun (809-833) established an academy at Baghdad, the House of Wisdom, which succeeded the Museum of Alexandria as the mathematical centre of its day. Here vital work was done in *translating* the existing classics (principally Greek and Hindu) into Arabic (including Euclid's *Elements* and Ptolemy's *Almagest*). Much of our knowledge of Greek and Hindu mathematics – e.g. Books V-VII of Apollonius' *Conics* – survived to the modern world only through Arabic transmission. Equally important, the Arabs were able to mingle the contributions from East and West, taking the best of both and synthesising them.

[4] Various transliterations are used; we follow Hourani.

[5] 1. The Qur'an, revealed to the Prophet by God in Arabic, is untranslatable, being the word of God; versions in other languages are termed interpretations.
2. The Prophet regarded followers of all three Abrahamic religions – Jews and Christians as well as Muslims – as peoples of the book, treating them with tolerance and respect, and Islam as the successor to the Judaeo-Christian tradition.

al-Kwarizmi (d. 850) *and algebra*

The leading Arab mathematician and astronomer, al-Kwarizmi, absorbed the mathematics of the Hindus. He wrote a book on Hindu numerals and their uses in calculation, which prenetrated Western civilisation so successfully that we still speak of 'Arabic' numerals 0,1,..,9; as mentioned under 'India' and taken for granted by al-Khwarizmi, these are of Hindu origin.

Aside on zero. The earliest known Hindu use of zero (0) in 876 is preceded by its use in a Muslim manuscript of 873. The Hindu word is sunya ('empty' or 'blank'), or sift ('empty') in Arabic, rendered as zephirum in Latin. Hence te English words cipher and zero, and the German Ziffer.

Algebra. Al-Kwarizmi's famous book *Al-Jabr wa al-Muqabala*[6] dealt with solutions of equations. Thus take e.g.

$$x^2 + 7x + 4 = 4 + 2x + 5x^3.$$

Al-jabr ('balancing' by transposing terms) gives

$$x^2 + 5x + 4 = 4 + 5x^3.$$

Al-muqabala ('simplifying') gives

$$x^2 + 5x = 5x^3.$$

As Arabic mathematics penetrated the West, the terms 'algiebar' and 'almachabel' were used in English in the 16th C., but shortened to *algebra*: in brief, the science of equations.

Algorithm. The term algorithm is a contraction of the name al-Khwarizmi, and is a tribute to the enormous importance of his work. The first algorithm one is likely to meet is the Euclidean algorithm for finding the hcf, but the concept grown greatly in importance with the use of the computer.

The most obvious figure with whom to compare al-Khwarizmi is Euclid. Euclid's *Elements* is not a particularly original work; rather it is a masterpiece of systematic elementary exposition. Similarly with al-Khwarizmi's 'Algebra': its handling of quadratic equations is anticipated by the Babylonians, by Diophantus and by Brahmagupta. But al-Khwarizmi was able to synthesise the three traditions – Mesopotamian, Greek and Hindu – and was the first to do so.

---

[6] Other transliterations include Al Khowarazmi (the name comes from that of a city in Persia), Al-jabr m'wa'l muquabalah.