

### M2PM3 PROBLEMS 2. 21.1.2010

Q1. (i) Plane polar coordinates  $(r, \theta)$  fail to be unique everywhere in the complex plane. Where does uniqueness fail, and why?

(ii) Similarly for spherical polar coordinates  $(r, \theta, \phi)$  on the sphere.

Q2. (i) Show that

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2} = \frac{\pi}{a} \quad (a > 0).$$

(ii) Show that

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)} = \frac{\pi}{ab(a + b)} \quad (a, b > 0).$$

Hint: use partial fractions to reduce to (i) if  $a \neq b$ . Then use this and continuity if  $a = b$ . We shall return to this example in Ch. III by residue calculus.

Q3. (i) Show that if

$$F(t) := \int_0^{\infty} e^{-x} \cos xt dx,$$
$$F(t) = 1/(1 + t^2).$$

Hint: integrate by parts twice.

(ii) Deduce that

$$\int_{-\infty}^{\infty} e^{ixt} \cdot \frac{1}{2} e^{-|x|} dx = \frac{1}{(1 + t^2)}.$$

NHB