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M2PM3 COMPLEX ANALYSIS

Professor N. H. BINGHAM, Spring 2011

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Course website: My homepage, link to Complex Analysis.

Recommended Student Texts:
John M. HOWIE, *Complex Analysis*, SUMS, 2003,
Hilary A. PRIESTLEY, *Introduction to Complex Analysis*, 2nd ed., OUP, 1990.
Robert B. ASH, *Complex Variables*, Academic Press, 1971 [available free online at Ash's website, University of Illinois]
Books for Reference:
Lars V. AHLFORS, *Complex Analysis*, 3rd ed., McGraw-Hill, 1979.
Walter RUDIN, *Real and Complex Analysis*, 2nd ed., McGraw-Hill, 1974.
For Real Analysis:
Walter RUDIN, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, 1976.
Library classmark: 517.53
Note the optional course M2PM5 *Metric Spaces and Topology*, given this term by Dr Thomas Sorensen.

Course Outline (33 lectures, 11 weeks, 3 lectures pw)
I. Preliminaries [9 lectures]
0. Why complex analysis?
1. Complex numbers
2. Preliminaries from Real Analysis and Topology
   1. Absolute and conditional convergence
   2. Uniform convergence
   3. Functions continuous on a closed interval
   4. Open and closed sets; metric spaces and topological spaces
   5. Infinite, countable and uncountable sets
   6. The Bolzano-Weierstrass theorem

7. Compactness; Heine-Borel theorem
8. Cauchy's General Principle of Convergence
9. O and o
10. Upper and lower limits
11. Power series

II. Holomorphic (Analytic) Functions: Theory [16 lectures]

1. Special complex functions
    1. Polynomials
    2. Exponentials
    3. Trigonometric functions
    4. Hyperbolic functions
    5. Logarithms
    6. Complex powers
2. Complex differentiability and the Cauchy-Riemann equations
3. Connectedness
4. Paths, line integrals, contours
5. Cauchy's Theorem
6. Cauchy's Integral Formulae
7. Cauchy-Taylor Theorem
8. Analytic continuation
    1. Power series. E.g., the geometric series
    2. Integrals. E.g., logarithms; the Gamma function $\Gamma(z)$
    3. Series. E.g., the Riemann zeta function $\zeta(s)$
    4. Identities. E.g., Euler's reflection formula $\Gamma(z)\Gamma(1-z) = \pi / \sin \pi z$
9. Maximum Modulus Theorem
10. Laurent's Theorem and singularities
11. Cauchy's Residue Theorem

III. Applications (Residue Calculus) [8 lectures].

1. Integration round the unit circle
2. Translation of the line of integration
3. Infinite integrals
4. Indentation
5. Branch points
6. Integrals involving many-valued functions
7. Summation of series
8. Expansion of a meromorphic function
    Infinite products for sin, cos and tan; Wallis' product