

**M2PM3 PROBLEMS 3. 25.1.2011**

Q1. Recall from lectures that (from de Moivre's theorem)  $\cos n\theta$  is a polynomial of degree  $n$  in  $\cos \theta$ :

$$\cos n\theta = T_n(\cos \theta).$$

Here  $T_n$  is called the  $n$ th *Tchebycheff polynomial* of the *first kind*.

Show that also

$$\sin n\theta = U_{n-1}(\cos \theta) \sin \theta,$$

where  $U_{n-1}$  is a polynomial of degree  $n-1$  – a Tchebycheff polynomial of the *second kind*.

Q2. Show that  $T_n$  has leading coefficient  $2^{n-1}$ .

Q3. Find  $T_3$ ,  $T_4$ ,  $T_5$ .

Q4. Show that, writing  $c, s, t$  for  $\cos \theta$ ,  $\sin \theta$ ,  $\tan \theta$ ,  $\tan n\theta$  is a rational function (ratio of polynomials) in  $t$ .

Q5. Show that the roots of the polynomial equation

$$7 - \binom{7}{3}t^2 + \binom{7}{5}t^4 - t^6 = 0$$

are  $\tan \pi/7$ ,  $\tan 2\pi/7$ , ...,  $\tan 6\pi/7$  (2010 Exam, Q1(ii)).

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