m3pm16cw.tex

M3PM16/M4PM16 ASSESSED COURSEWORK 2012

Set Th 23 February (Week 7) ; deadline 12 noon Th 8 March (Week 9)

Q1. By considering the Dirichlet series of

$$-\frac{\zeta'(s)}{\zeta^2(s)} = \frac{1}{\zeta(s)}\cdot\frac{-\zeta'(s)}{\zeta(s)} = \frac{d}{ds}\Big(\frac{1}{\zeta(s)}\Big)$$

and equating coefficients, or otherwise, show that if

$$N(x) := \sum_{n \leq x} \mu(n)\log n,$$

the Prime Number Theorem (in the form $\psi(x) \sim x$) implies

$$N(x) = o(x\log x).$$

(You may use the Proposition in II.5.)

Q2. If

$$M(x) := \sum_{n \leq x} \mu(n),$$

show that the Prime Number Theorem (again in the form $\psi(x) \sim x$) implies

$$M(x) = o(x).$$

(You may use Q1.)

NHB