

DRAMATIS PERSONAE: WHO DID WHAT WHEN

- N. H. Abel (1802-1829): Abel's Lemma, Abel's test, Abel's summation formula, I.3; Abel's continuity theorem, II.1
- Salomon Bochner (1899-1982): History of FTA, I.1
- William, Lord Brouncker ((1620-1684), in 1655: continued fraction for π
- A. L. Cauchy (1789-1857): pioneered Complex Analysis, 1825-9, I.2, III.1; Cauchy product, I.6
- P. L. Chebyshev (Tschebyschef in [L]) (1821-1894) in 1849-51: bounds on $\pi(x)/li(x)$, and if it has a limit, the limit is 1, III.1; Chebyshev's Upper and Lower Estimates, III.2; proof of Bertrand's postulate (Problems 8)
- Richard Dedekind (1831-1916) in 1872: Dedekind cuts, or sections, for construction of the reals, I.1; Dedekind cuts for σ_c , σ_a , II.1; Galileo-Dedekind definition of an infinite set, III.10.5
- L. E. Dickson (1874-1954): History of Number Theory, 1919-23, I.1
- J. P. G. Lejeune Dirichlet (1805-1859): Dirichlet series, 1838, 1839, II; Dirichlet's test, I.3; Dirichlet convolution, II.3; Dirichlet eta function (alternating zeta function), II.1; Dirichlet's Hyperbola Identity, II.9; primes in an AP, 1837, III.10.5; Dirichlet's Pigeonhole Principle, 1837, III.10.5
- Paul Erdős (1913-1996): Erdős-Selberg elementary proof of PNT in 1949, III.1, III.10; Erdős-Kac CLT in 1939, III.10.1; proof of Bertrand's postulate in 1932, Problems 8.
- Euclid of Alexandria, c. 300 BC: Euclid's Elements, esp. Books VI, IX, X; infinitely many primes, I.1, II.4
- Leonhard Euler (1707-1783): notation for e and $e^{i\pi} + 1 = 0$ in 1727; integral definition of $\Gamma(\cdot)$ in 1729, I.7; Euler's constant γ in 1734, I.4; $\zeta(2) = \pi^2/6$ in 1735; Euler's summation formula, I.9, and Euler-Maclaurin summation formula in 1736, I.4, III.3; ζ for real arguments, II.1, and Euler products in 1737, II.4; $\zeta(2n)$ in terms of π^{2n} and Bernoulli numbers in 1739; Euler's totient function ϕ in 1760, II.6
- C. F. Gauss (1777-1855): Disquisitiones Arithmeticae and Fundamental Theorem of Arithmetic (FTA), 1801, I.1; conjectured PNT (c. 1799), III.1
- Jacques Hadamard (1865-1963) in 1896: Proof of PNT, III.1
- G. H. Hardy (1877-1947): Hardy and Wright; Hardy-Ramanujan LLN for ω , Ω in 1917, III.10.1; equivalent of PNT, III.10.4; *Divergent Series* in 1949
- C. Hermite (1822-1901) in 1873: e is transcendental
- Shikao Ikehara (1904-1984) in 1931: Wiener-Ikehara theorem, III.7; equivalence of PNT and non-vanishing of $\zeta(1 + it)$, III.10.4

A. E. Ingham (1900-67): Cambridge Tract *The distribution of prime numbers* in 1932; Ingham-Newman Tauberian theorem in 1935, III.6
 Mark Kac (1914-1984): Erdős-Kac CLT in 1839, III.10.1; Kac's Dictum, Primes play a game of chance, in 1959, III.10.1
 N. M. Korobov (1917-) in 1958: error term in PNT, III.10.2; zero-free region for ζ , III.10.2
 Edmund Landau (1877-1938): *Handbuch* in 1909; Poisson extension of PNT in 1900, III.9; PNT equivalent to $M(x) = o(x)$ in 1912, Enhanced CW.
 Henri Lebesgue (1875-1941) in 1902: Lebesgue integral, Riemann-Lebesgue Lemma, I.7
 A. M. Legendre (1752-1833) in 1798: conjectured PNT, III.1
 C. L. F. Lindemann (1852-1939) in 1882: π is transcendental
 J. E. Littlewood (1885-1977) in 1911: Littlewood's O -Tauberian theorem, III.7
 H. C. F. von Mangoldt (1854-1925): von Mangoldt function Λ in 1894, II.6; $\sum_1^\infty \mu(n)/n = 0$ in 1898, III.8, III.10.4
 F. Mertens (1840-1927): Multiplication of series, I, Handout; Mertens' First and Second Theorems and Mertens' formula in 1874, II.7
 A. F. Möbius (1790-1868) in 1831: Möbius function μ , Möbius Inversion Formula II.5
 G. Morera (1856-1909) in 1889: Morera's theorem, I.2
 John von Neumann (1903-57) in 1923: Construction of \mathbf{N} , I.1
 D. J. Newman (1930-1967) in 1980: Ingham-Newman Tauberian theorem, III.6; Newman's proof of PNT, III.8
 Sir Isaac Newton (1642-1727) in 1665: e , the base of natural logarithms
 Oskar Perron (1880-1975) in 1908: Perron's formula (inversion formula for the Mellin transform), III.5
 S. Ramanujan (1887-1920): Hardy-Ramanujan LLN for ω, Ω in 1917, III.10.1
 G. F. Bernhard Riemann (1826-1866): Riemann integral in 1854, I.7; Riemann-Lebesgue Lemma, 1.7; Riemann zeta function $\zeta(s)$ in 1859, II.1, III.1; Riemann Hypothesis (RH) in 1859, III.1; functional equation for ζ in 1859, III.3
 Atle Selberg (1917-2009) in 1939: Elementary proof of PNT, III.1, III.10
 Paul Turán (1910-1976) in 1950 and book of 1976: error terms of PNT and zero-free regions of ζ , III.10.3
 C. de la Vallée Poussin (1866-1962) in 1896: Proof of PNT, III.1
 R. C. Vaughan (1945 -): M3PM16, L0; Introduction to 2nd ed. (1990) of Ingham's 1932 Cambridge Tract; Vaughan's Dictum, III.10.1
 I. M. Vinogradov (1891-1983) in 1958: error term in PNT, III.10.2; zero-free

region for ζ , III.10.2

Karl Weierstrass (1815-1897): Product definition of $\Gamma(z)$, I.8

Norbert Wiener (1894-1964) in 1932: Wiener-Ikehara theorem, III.7; Wiener's (general) Tauberian theorem, III.10.3

E. M. Wright (1906-2005): Hardy and Wright [HW]; Wright's proof of Landau's Poisson PNT in 1954, III.9

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