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Lecture 0. 10.1.2012.

M3PM16/M4PM16 ANALYTIC NUMBER THEORY

Professor N. H. BINGHAM, Spring 2012

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Tuesdays 9-11, 340 and Thursdays 5-6, 340

Course website: My homepage, link to M3PM16; Office hour, Thursdays 4-5

Recommended student text:
[J] G. J. O. JAMESON, *The Prime Number Theorem*, LMS Student Texts 53, CUP, 2003, Ch. 1-3.
Alternative student text:
[A] T. M. APOSTOL, *Introduction to analytic number theory*, UTM, Springer, 1976, Ch. 1-4, 11, 13.
References:
[N] D. J. NEWMAN, *Analytic number theory*, GTM 177, Springer, 1998.
[R] H. E. ROSE, *A course in number theory*, OUP, 1988.
[HW] G. H. HARDY and E. M. WRIGHT, *An introduction to the theory of number*, 5th ed., OUP, 1979 (or 6th ed., with D. R. Heath-Brown and J. H. Silverman, 2008).
[L] E. LANDAU, *Handbuch der Lehre von der Verteilung der Primzahlen*, 2nd ed., Chelsea, New York, 1953 (1st ed., Teubner, 1909).
Complex Analysis:
We shall make extensive use of Complex Analysis. Our reference here will be the course website for M2PM3 (which I taught you all!)

Course Outline (30 lectures, 10 weeks, 3 lectures pw)
I. Preliminaries [5 lectures]
1. Primes [L1]
2. Limits of holomorphic functions [L2]
3. Abel (= partial) summation [L2, L3]
4. The integral test and Euler's constant [L3]
5. Infinite products [L4]

6. The Riemann-Lebesgue Lemma [L4]
7. The Gamma function [L5]
8. Euler's summation formula [L5]

II. Arithmetic functions and Dirichlet series [$9\frac{1}{2}$ lectures]

1. Dirichlet series; the Riemann zeta function $\zeta(s)$ [L6, L7]
2. Holomorphy [L8]
3. Convolutions [L8, L9]
4. Euler products [L9, L10]
5. The Möbius function μ [L10, L11]
6. More special Dirichlet series. The von Mangoldt function Λ [L11, L12]
7. Mertens' theorems [L12, L13, L14]
8. The prime divisor functions [L14]
9. Dirichlet's Hyperbola Identity [L15]

III. The Prime Number Theorem (PNT) and its relatives [$15\frac{1}{2}$ lectures]

1. PNT [L15, L16]
2. Chebyshev's theorems [L16, L17, L18]
3. Continuation of ζ [L19]
4. Non-vanishing of $\zeta(1 + it)$ [L20]
5. Perron's formula [L21, L22, L23]
6. The Ingham-Newman Tauberian theorem [L23, L24]
7. An elementary Tauberian theorem [L25, L26]
8. Proof of PNT [L26]
9. Landau's Poisson extension of PNT: Primes play a game of chance [L27, L28]
10. Complements [L29, L30]

Dramatis Personae: Who did what when

Exam and Coursework

The exam will be in standard format for M3PM/M4PM courses (4 questions, 20 marks each). There will be one Assessed Coursework for all (Week 7), and one Enhanced Coursework for M4PM16 (Week 10).

This is both an old course (Imperial College used to be a noted centre for Analytic Number Theory, when this course was standard) and a new course (it has not been taught here since Professor R. C. Vaughan and Dr W. W. Chen left many years ago). So I shall set a Mock Exam, with Solutions.

Website

I shall set Problems and Solutions 1-8 weekly, and post them on the website. As with M2PM3: lectures will be delivered on the whiteboard, but lecture notes in TeX will appear on the website. NHB, 10.1.2012