

M3PM16/M4PM16 PROBLEMS 1. 17.1.2013

Q1. Write

$$li(x) := \int_2^x \frac{dt}{\log t}$$

(the *logarithmic integral*, which we shall meet in Ch. II, III in the Prime number Theorem, PNT), show that

$$li(x) \sim \frac{x}{\log x} \quad (x \rightarrow \infty)$$

(i.e. LHS/RHS $\rightarrow 1$).

Q2. For $m = 1, 2, \dots$, show that

$$li(x) - \left(\frac{x}{\log x} + \frac{1!x}{\log^2 x} + \dots + \frac{(m-1)!x}{\log^m x} \right) \sim \frac{m!x}{\log^{(m+1)} x}$$

(repeated integration by parts, or otherwise).

Q3. For p_n the n th prime, show that PNT

$$\pi(x) := \sum_{p \leq x} 1 \sim \frac{x}{\log x}$$

gives

$$p_n \sim n \log n$$

(indeed, this is equivalent to PNT).

Q4. Show that PNT further gives

$$p_n = n(\log n + \log \log n + O(1)).$$

NHB