m3pm16prob5.tex

M3PM16/M4PM16 PROBLEMS 5. 14.2.2013

Q1. By considering the series expansion of $-\log(1-x)$, or otherwise, show that $\prod(1-1/p)$ diverges.

Q2. Use the divergence of $\prod(1-1/p)$ to show (by considering the number $N(x,r)$ of $n \leq x$ not divisible by any of the first r primes p_k, or otherwise) that

$$\pi(x) = o(x).$$

(This bound is much weaker than PNT $\pi(x) \sim li(x) \sim x/\log x$, but is useful and non-trivial.)

Q3. Show that if $c := a * b$ and b are multiplicative, then a is multiplicative.

NHB