m3pm16l0.tex
Lecture 0. 14.1.2014.

M3PM16/M4PM16 ANALYTIC NUMBER THEORY

Professor N. H. BINGHAM, Spring 2014

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Tuesdays 5-6 and Fridays 9-10 and 5-6, 340

Course website: My homepage, link to M3PM16

Office hour, Fridays 4-5

Recommended student text:
[J] G. J. O. JAMESON, *The Prime Number Theorem*, LMS Student Texts 53, CUP, 2003, Ch. 1-3 and 5.

Alternative student texts:
[HW] G. H. HARDY and E. M. WRIGHT, *An introduction to the theory of number*, 5th ed., OUP, 1979 (or 6th ed., rev. D. R. Heath-Brown and J. H. Silverman, 2008).
[A] T. M. APOSTOL, *Introduction to analytic number theory*, UTM, Springer, 1976, Ch. 1-4, 11, 13.

References:
[MV] H. L. MONTGOMERY and R. C. VAUGHAN, *Multiplicative number theory: I. Classical theory.* Cambridge studies in adv. math. 97, CUP, 2007.
[T] E. C. TITCHMARSH, *The theory of the Riemann zeta-function*, 2nd ed. (rev. D. R. Heath-Brown), OUP, 1986 (1st ed. 1951).
[N] D. J. NEWMAN, *Analytic number theory*, GTM 177, Springer, 1998.
[R] H. E. ROSE, *A course in number theory*, OUP, 1988.
[L] E. LANDAU, *Handbuch der Lehre von der Verteilung der Primzahlen*, 2nd ed., Chelsea, New York, 1953 (1st ed., Teubner, 1909).
[D] H. DAVENPORT, *Multiplicative number theory*, 3rd ed. (rev. H. L. Montgomery), Grad. Texts Math. 74, Springer, 2000 (1st ed. 1967, 2nd 1980).

[A2] T. M. APOSTOL, *Modular functions and Dirichlet series in number theory*. Grad. Texts in Math. 41, Springer, 1976.

Complex Analysis: See the link to M2PM3 (2011) on my homepage, or
[T2] E. C. TITCHMARSH, *Theory of functions*, 2nd ed., OUP, 1039 (1st ed. 1932).
[Ahl] L. V. AHLFORS, *Complex analysis*, 3rd ed., McGraw-Hill, 1979 (1st ed. 1953, 2nd ed. 1966).
[K] J. KOREVAAR, *Tauberian theory: A century of developments*. Grundl. math. Wiss. **329**, Springer, 2004.
[WW] E. T. WHITTAKER & G. N. WATSON, *Modern analysis*, 4th ed., CUP, 1927/1946.
Fourier Analysis: We use Poisson's summation formula. See e.g.
[AL] Professor Ari Laptev's homepage, link to M2PM3 (2012), last chapter;
[Kat] Y. KATZNELSON, *An introduction to harmonic analysis*, 3rd ed., CUP, 2004, §1.15.

Course Outline (33 lectures, 11 weeks, 3 lectures pw)

I. Preliminaries [5 lectures]
1. Primes [L1]
2. Limits of holomorphic functions [L2]
3. Abel (= partial) summation [L2-3]
4. The integral test and Euler's constant [L3]
5. Infinite products [L4]
6. The Riemann-Lebesgue Lemma [L4]
7. The Gamma function [L5]
8. Euler's summation formula [L5]

II. Arithmetic functions and Dirichlet series [9 lectures]
1. Dirichlet series [L6]
2. The Riemann zeta function $\zeta(s)$ [L7]
3. Holomorphy [L8]
4. Convolutions [L8-9]
5. Euler products [L9-10]
6. The Möbius function μ [L10-11]
7. More special Dirichlet series. The von Mangoldt function Λ [L11-12]

8. Mertens' theorems [L12-14]
9. Dirichlet's Hyperbola Identity [L14]

III. The Prime Number Theorem (PNT) and its relatives [10 lectures]
1. PNT [L15]
2. Chebyshev's theorems [L16-18]
3. Analytic continuation of ζ [L19]
4. Non-vanishing on the 1-line: $\zeta(1+it) \neq 0$ [L20]
5. Newman's theorem [L21-23]
6. Proof of PNT [L23]
7. The functional equation for the Riemann zeta function [L23-4]

IV. PNT with Remainder [9 lectures]
1. Perron's formula [L25-27]
2. Further Complex Analysis [L27-29]
3. The zero-free region [L29-30]
4. Bounds for $-\zeta'/\zeta$ [L30-31]
5. Proof of PNT with Remainder [L31-32]
6. Equivalents to PNT [L33]
Dramatis Personae: Who did what when

Exam and Coursework. The exam will be in standard format for M3PM/M4PM courses (4 questions, 20 marks each); similarly for the Assessed Coursework and Mastery Question.
Website. I shall set Problems and Solutions weekly, and post them on the website. As with M2PM3: lectures will be delivered on the whiteboard, but lecture notes in TeX will appear on the website.

This year. All the 2013 material is on the website (link to 'Last year's course'); the 2014 material will go up as delivered.

The course was re-introduced in 2012 (after a long gap); I then followed Ch. 1-3 of Jameson's book closely, proving PNT twice, without remainder term. In 2013 I proved PNT once without remainder (by the Wiener-Ikehara theorem and Fourier Analysis) and once with (by Complex Analysis: distribution of zeta zeros and the Riemann-von Mangoldt formula). This year, I again prove PNT once without remainder term (by Newman's method), and once with (by Perron's formula (IV.1) and the Borel-Carathéodory theorem (IV.2)). We follow 2013 for L1-20, but then diverge. NHB, 14.1.2014