m3pm16prob1.tex

M3PM16/M4PM16 PROBLEMS 1. 17.1.2014

Q1. (i) Show that a real number x is rational iff its decimal expansion terminates or recurs.
(ii) What can be said about the decimal expansion of m/n (cancelled down to its lowest terms)?
(iii) What about binary expansions? ternary? etc.

Q2. Write

$$li(x) := \int_2^x \frac{dt}{\log t}$$

(the *logarithmic integral*, which we shall meet in Ch. II, III in the Prime number Theorem, PNT), show that

$$li(x) \sim \frac{x}{\log x} \qquad (x \to \infty)$$

(i.e. LHS/RHS $\to 1$).

Q3. For $m = 1, 2, \ldots$, show that

$$li(x) - (\frac{x}{\log x} + \frac{1!x}{\log^2 x} + \ldots + \frac{(m-1)!x}{\log^m x}) \sim \frac{m!x}{\log^{(m+1)} x}$$

(repeated integration by parts, or otherwise).

Q4. For p_n the nth prime, show that PNT

$$\pi(x) := \sum_{p \le x} 1 \sim \frac{1}{\log x}$$

gives

$$p_n \sim n \log n$$

(indeed, this is equivalent to PNT).

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Q4. Show that PNT further gives

$$p_n = n(\log n + \log\log n + O(1)).$$

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