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Lecture 0. 13.1.2015.

M3PM16/M4PM16 ANALYTIC NUMBER THEORY

Professor N. H. BINGHAM, Spring 2015

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Tuesdays 5-6, 140, Wed 9-10, 340 and Thur 2-3, 139

Course website: My homepage, link to M3PM16

Office hour, Thursdays, 3-4

Recommended student text:
[J] G. J. O. JAMESON, *The Prime Number Theorem*, LMS Student Texts 53, CUP, 2003, Ch. 1-3 and 5.

Alternative student texts:
[HW] G. H. HARDY and E. M. WRIGHT, *An introduction to the theory of number*, 5th ed., OUP, 1979 (or 6th ed., rev. D. R. Heath-Brown and J. H. Silverman, 2008).
[A] T. M. APOSTOL, *Introduction to analytic number theory*, UTM, Springer, 1976, Ch. 1-4, 11, 13.

References:
[Ten] G. TENENBAUM, *Introduction to analytic and probabilistic number theory*, 2nd ed. Cambridge studies in adv. math. 46, CUP, 1995.
[MV] H. L. MONTGOMERY and R. C. VAUGHAN, *Multiplicative number theory: I. Classical theory.* Cambridge studies in adv. math. 97, CUP, 2007.
[T] E. C. TITCHMARSH, *The theory of the Riemann zeta-function*, 2nd ed. (rev. D. R. Heath-Brown), OUP, 1986 (1st ed. 1951).
[N] D. J. NEWMAN, *Analytic number theory*, GTM 177, Springer, 1998.
[R] H. E. ROSE, *A course in number theory*, OUP, 1988.
[L] E. LANDAU, *Handbuch der Lehre von der Verteilung der Primzahlen*, 2nd ed., Chelsea, New York, 1953 (1st ed., Teubner, 1909).
[D] H. DAVENPORT, *Multiplicative number theory*, 3rd ed. (rev. H. L.

Montgomery), Grad. Texts Math. 74, Springer, 2000 (1st ed. 1967, 2nd 1980).
[A2] T. M. APOSTOL, *Modular functions and Dirichlet series in number theory*. Grad. Texts in Math. 41, Springer, 1976.

Complex Analysis: We shall make heavy use of this, in particular of the wonderful technique of *analytic continuation*. For background, see e.g. the link to M2PM3 (2011) on my homepage (analytic continuation: L22, 23).
References:
[T2] E. C. TITCHMARSH, *Theory of functions*, 2nd ed., OUP, 1039 (1st ed. 1932) (*Borel-Caratheodory theorem*, §5.5).
[Ahl] L. V. AHLFORS, *Complex analysis*, 3rd ed., McGraw-Hill, 1979 (1st ed. 1953, 2nd ed. 1966).
[K] J. KOREVAAR, *Tauberian theory: A century of developments*. Grundl. math. Wiss. **329**, Springer, 2004.
[WW] E. T. WHITTAKER & G. N. WATSON, *Modern analysis*, 4th ed., CUP, 1927/1946 (Ch. XII: The Gamma function; Ch. XIII: The Riemann zeta function).
Fourier Analysis: We use Poisson's summation formula. See e.g.
[AL] Professor Ari Laptev's homepage, link to M2PM3 (2012), last chapter;
[Kat] Y. KATZNELSON, *An introduction to harmonic analysis*, 3rd ed., CUP, 2004, §1.15.

Course Outline (33 lectures, 11 weeks, 3 lectures pw)

I. Preliminaries [5 lectures]
1. Primes [L1]
2. Limits of holomorphic functions [L2]
3. Abel (= partial) summation [L2-3]
4. The integral test and Euler's constant [L3]
5. Infinite products [L4]
6. The Riemann-Lebesgue Lemma [L4]
7. The Gamma function [L5]
8. Euler's summation formula [L5]

II. Arithmetic functions and Dirichlet series [9 lectures]
1. Dirichlet series [L6]

2. The Riemann zeta function $\zeta(s)$ [L7]
3. Holomorphy [L8]
4. Convolutions [L8-9]
5. Euler products [L9-10]
6. The Möbius function μ [L10-11]
7. More special Dirichlet series. The von Mangoldt function Λ [L11-12]
8. Mertens' theorems [L12-14]
9. Dirichlet's Hyperbola Identity [L14]

III. The Prime Number Theorem (PNT) and its relatives [10 lectures]
1. PNT [L15]
2. Chebyshev's theorems [L16-18]
3. Analytic continuation of ζ [L19]
4. Non-vanishing on the 1-line: $\zeta(1+it) \neq 0$ [L20]
5. Newman's theorem [L21-23]
6. Proof of PNT [L23]
7. The functional equation for the Riemann zeta function [L23-4]

IV. PNT with Remainder [9 lectures]
1. Perron's formula [L25-27, $2\frac{1}{2}$ h]
2. Further Complex Analysis [L27-29, 2h]
3. The zero-free region [L29-30, 1h]
4. Logarithmic bound for $-\zeta'/\zeta$ [L30-31, $1\frac{1}{4}$h]
5. Proof of PNT with Remainder [L31-32, $1\frac{1}{4}$h]
6. Equivalents to PNT [L33, 1h]

Dramatis Personae: Who did what when

Exam and Coursework. The exam will be in standard format for M3PM/M4PM courses (4 questions, 20 marks each); similarly for the Assessed Coursework and Mastery Question.
Website. I shall set Problems and Solutions weekly, and post them on the website. As with M2PM3: lectures will be delivered on the whiteboard, but lecture notes in TeX will appear on the website.

This year. I will follow 2014 fairly closely. See the link to 'Last year's course' for a sneak preview, and the evolution of the course since its re-introduction in 2012.